Trade-offs Between Maximal Forward Gain and Minimal Backward Gain of a Yagi Antenna

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***Abstract*—Optimization of Yagi antenna for maximum forward gain and minimal backward gain is conducted. Possible trade-offs between objectives are presented in the form of a Pareto front. It is determined by weighting the cost functions with many different combinations of weighting factors. Antenna solutions are generated by genetic algorithm and evaluated in method of moments based numerical solver, in a frequency range of interest. Our main objective is finding the best compromises between forward and backward gain of Yagi antenna, using the outlined optimization process.**

***Index Terms*—backward gain, forward gain, genetic algorithm, Pareto front, Yagi antenna**

I. Introduction

An optimal design of Yagi antenna that meets required criteria (eg. maximum forward gain, minimum backward gain, maximum front-to-back ratio of gain etc.) is practically impossible to find with analytical methods. Studies of impact of lengths and spacing of elements on the radiation performance leads to a complex nonlinear optimization problem, due to fact that parasitic elements are strongly coupled through electromagnetic (EM) field. With the advancement of the computers and numerical EM algorithms, the optimization has become common approach for design of Yagi antenna.

In earlier research, different optimization techniques based on adjusting elements' lengths and distances are developed in order to exploit maximum forward gain for Yagi antenna [1-3]. As concluded in [2], there exist many local minima in the optimization space, which is defined by the total number of optimization variables and their predefined (given) ranges. Since the introduction of Genetic Algorithm (GA) [4], many researchers utilized its standard form to solve the problem of optimization of Yagi antenna [5-8]. Stochastic GA operators allow the algorithm to find the global minimum in the given optimization space, thus making it a good candidate for this optimization problem.

Having more than one objective leads to a multiobjective optimization, which is intrinsically more complex than the single criterion optimization. In that case, theoretically there is no single best solution, but rather there is a set of Pareto optimal solutions (i.e., a set of the best possible trade-offs among specified criteria). Pareto front for the forward gain maximization and backward gain minimization is estimated using GA and presented in this paper. Note that the side lobes in the radiation pattern are not considered. Although it might seem that maximization of the forward gain and minimization of the backward gain are the same optimization goal, the numerical results show that these objectives are conflicting. This statement is in agreement with results presented in [9], where a different method is used to estimate Pareto front.

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II. Yagi Antenna and Its Numerical EM Model

The analyzed twelve-element Yagi antenna consists of a driven element, a single reflector and ten directors [10]. The antenna is designed to operate at the central frequency of 300 MHz, and the frequency range of interest is 295 MHz to 305 MHz. EM analysis is performed in WIPL-D software package [11] that is method of moments (MoM) based numerical solver, with higher order basis functions. The optimization is done in an external application, created for the purpose of the presented work. Every element of the antenna is modeled as a wire, forcing kernel to use a thin-wire approximation, which is very fast and sufficiently accurate for the thin wires [8]. The main reason for using thin-wire approximation is to speed up the numerical EM analysis. In every iteration of the optimization, a different set of antenna parameters is provided to WIPL-D kernel for the numerical EM analysis. After the simulation, the obtained output results for antenna gain in forward and backward directions are used to evaluate the cost function used in the optimization. The total number of unknowns in MoM matrix goes up to 36, where the limit is established from the case when the lengths of elements take the highest values from the ranges of optimization variables. With nowadays computers, this is a relatively small-size numerical problem that results in acceptably fast numerical analysis (typically 0.2 s per simulation). The used desktop computer configuration consists of Intel® Core™ i7 CPU 950@3.07 GHz and 24 GB of RAM.

The optimization variables are lengths and spacings between the elements of the Yagi antenna [10]. All the dimensions are chosen from the interval from 0.2 λ to 0.8 λ, where λ is a free-space wavelength at 300 MHz. Lengths of each director, as well as spacings between the adjacent directors, are kept the same, resulting in the total of six optimization variables. The two considered optimization criteria are: the highest possible gain in the forward direction, and the minimal possible gain in the backward direction.

III. Algorithm Used for Multiobjective Optimization

From a general perspective, every optimization algorithm consists of the following steps: creating a starting solution or groups of solutions, evaluating the cost functions of the current solution(s), checking whether some of the solutions satisfies given criteria, creating a new set of solutions and repeating the process. The optimization stops either if the goal has been achieved or the maximum allowed number of iterations has been attained. In the first case, the solution of the problem is provided to the user. In the second case, the best found solution is provided to the user.

In the problem of finding trade-offs between forward and backward gain of Yagi antenna we are looking for the best possible solution that can be found for the given number of iterations.

The fitness functions for these two objectives are defined as follows:

$$f_1 = A - G_{\text{front}}\,, \tag{1}$$

$$f_2 = B + G_{\text{back}}\,. \tag{2}$$

Here G_{front} denotes gain in dBi in the forward direction, and G_{back} denotes gain in dBi in the backward direction. G_{front} is expected to be positive [10], while G_{back} should be negative for a good solution from an engineering point of view. The variables A and B are positive numbers chosen to be large enough to ensure strictly positive values for both cost functions in all possible cases in the optimization problem. Moreover, a proper choice of the constants makes different criteria comparable in terms of their (numerical) significance, relative one to the other. Combining these two criteria into a single cost function, for the given multicriteria optimization problem, is obtained by summation of (1) and (2) as

$$f = f_1 + f_2\,. \tag{3}$$

Considering the forms of f_1 and f_2 together with the optimization goals, it is obvious that a lower cost function always points out to a better solution. Formula (3) is calculated in N equidistant frequency points, so the final cost function is expressed as a root mean square deviation:

$$F = \sqrt{\frac{\sum_{i=1}^{N} f_i^2}{N}}\,, \tag{4}$$

where f_i stands for value of f at i-th simulation frequency. Note that in the case of Yagi antenna optimization we use $N = 5$.

A. Genetic Algorithm

The standard form of GA has been adapted to this problem according to [10]. An individual here refers to a different antenna design solution, described with its variables that are called genes in the GA terminology. The initial population was generated randomly, using uniform random generator.

In the process of the selection, the quality of each solution is examined by evaluating the fitness function (4). Selection operator is realized using tournament selection. In every round, pairs of individuals are randomly chosen to compete in "duels", and the ones that fit better (i.e., the ones that have lower cost functions) qualify for the next round of the tournament. The tournament ends when the number of not eliminated individuals drops to initially defined number of survivors k. Survivors are used to create solutions ("offspring") for the next generation. Although the best solutions are being forced in average through the tournament selections, there is a good chance that a solution that might not be in the fittest k (if we would sort the whole population at once), becomes chosen for crossover just because of having randomly chosen less-fit opponents. This effect is intentional and its purpose is the preserving of certain amount of diversity among the solutions. This is what makes tournament selection significantly different and more applicable in comparison to the pure elitism, which allows only the best individuals to take a part in crossovers.

Once the selection is finished, a crossover is performed. The crossover is a process of recombination parents' genes (optimizing variables), in order to produce solutions for the next generation. Two parents are randomly chosen to create three descendants according to formulas (5), (6) and (7), with predefined probability of the crossover $P_c = 0.8$, as in [10]. In other words, this probability represents chances that the two individuals will be used for crossover after being chosen, and will not be discarded.

$$\mathbf{d}_1 = \mathbf{p}_2 + \alpha(\mathbf{p}_1 - \mathbf{p}_2) \tag{5}$$

$$\mathbf{d}_2 = \mathbf{p}_2 - \alpha(\mathbf{p}_1 - \mathbf{p}_2) \tag{6}$$

$$\mathbf{d}_3 = \mathbf{p}_1 + \alpha(\mathbf{p}_1 - \mathbf{p}_2) \tag{7}$$

Here **d** and **p** represent descendant's and parent's chromosomes respectively, both defined as a vector of optimization variable (genes), while α is a real number, randomly generated from the interval [0,1] for every crossover. Crossover stops when the number of new solutions are created so that the whole next generation is populated. In attempt to avoid possible multiple convergences to the same local minima in the optimization space, not a single solution from the previous generation is allowed to be used in the next generation.

The mutation operator is realized as a replacement of genes with the randomly chosen values with uniform distribution in the range between the lower and the upper bound for the selected variable. For each gene of each generated solution, mutation is to be executed with the probability $P_m = 0.15$. The controlled mutation causes the other parts of optimization space to be explored, which may eventually lead to a better solution. Increasing the probability of mutation degenerates even good solutions, making it more difficult or impossible for GA to converge.

B. Pareto Front

In cases of multiobjective problems, there is rarely a solution that dominates all the other solutions with regard to all the criteria. Most often the criteria are conflicting, so it is not possible to make an improvement in one objective without deterioration the others. Pareto front is the set of the best possible trade-offs that can be theoretically achieved, and it is defined by an infinite number of Pareto-optimal solutions, if the optimization space is continuous and if the criteria are defined as real-number functions. A solution is pareto optimal if there is no other solution in the search space that has better performances for all the given objectives [10], [12]. Pareto front provides deep insight into compromises that should be made for the sake of overall performance, hence one can choose the most suitable solution for a problem under consideration.

The aim of this work is to present the best found compromises between the maximum gain in the forward direction and the minimum gain in the backward direction for Yagi antenna described in Section II. The method starts with weighting the penalties for cost functions (1) and (2), thus increasing the importance of one or the other criterion in the cost function. Now the total cost function becomes:

$$f_{\text{tot}} = w_1 f_1 + w_2 f_2 \qquad (8)$$

By replacing f_{tot} in (4) with expression from (8), the final cost function used in the optimization yields to:

$$F_{\text{tot}} = \sqrt{\frac{\sum_{i=1}^{N} (w_1 f_{1,i} + w_2 f_{2,i})^2}{N}}\,, \qquad (9)$$

where $f_{1,i}$ and $f_{2,i}$ denote values of (1) and (2) on i-th frequency.

In order to decrease the cost function defined above, the optimization will tend to produce solutions which minimize the cost function that is associated with higher weighting coefficient, relative to the ratio of (1) and (2). Roughly speaking, the ratio of the separate cost functions in the final solution is expected to be proportional to the ratio of their weighting coefficients. From that perspective, it is clear that A and B in the definitions for f_1 and f_2 should be chosen carefully, since they directly influence the final cost function. It should also be noted that values of the forward and the backward gain that are summed here, significantly differ for various antennas.

In order to determine the Pareto front accurately, the optimization needs to be performed for all possible combination of w_1 and w_2 [12]. The theoretical number of those combinations is infinite. For that reason, only few extreme scenarios are enough to get an engineering insight into the distribution of the optimal solutions, i.e., to estimate the Pareto front.

IV. Antenna Optimization and Results

The parameters used in GA are as follows: the population has 16 solutions, 4 selected solutions from the population creates the next generation, the total number of generations is 100. One iteration is an evaluation of the cost function, so there are 1600 iterations in a single optimization run, i.e., to find the cost functions for all solutions in one generation of GA. The variable A in f_1 is chosen to be 20 dBi, and B in f_2 is chosen to be 50 dBi. These values are determined to be related to the gain of the best-found solutions when the algorithm was run solely for the front and the back gain, as shown in Fig. 1. and Fig. 2. The total number of frequency points in EM analysis and the optimization is 5. The sets of weighting coefficients used for finding the Pareto front are (w_1=0, w_2=1), (w_1=1, w_2=0), (w_1=1, w_2=1), (w_1=1, w_2=2), (w_1=2, w_2=1), (w_1=1, w_2=3), (w_1=3, w_2=1), (w_1=1, w_2=5), (w_1=5, w_2=1), (w_1=1, w_2=10), (w_1=10, w_2=1), (w_1=1, w_2=100), (w_1=100, w_2=1). For each pair of weighting factors, GA is restarted 10 times, and the best-found solutions from every run are presented as the results.

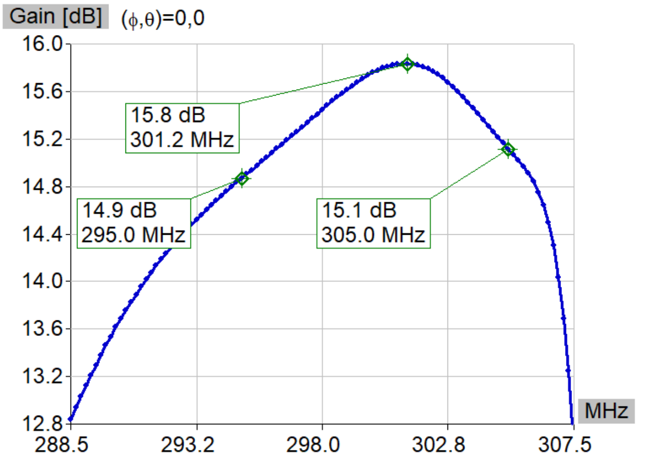

Fig. 1. Gain in forward direction in terms of frequency, obtained for (w_1=1, w_2=0)

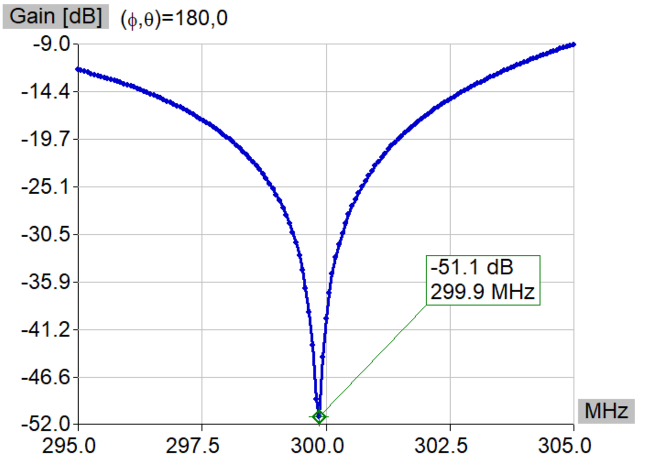

Fig. 2. Gain in backward direction in terms of frequency, obtained for (w_1=0, w_2=1)

Two new variables, x and y, are presented in Fig. 3 and they are defined as root-mean-square of the forward and backward gains as:

$$x = \sqrt{\frac{\sum_{i=1}^{N} (A - G_{\text{front},i})^2}{N}} \qquad (9)$$

$$y = \sqrt{\frac{\sum_{i=1}^{N} (B + G_{\text{back},i})^2}{N}}\,, \qquad (10)$$

where $G_{\text{front},i}$ and $G_{\text{back},i}$ stand for values of gain on i-th frequency, and N = 5.

It can be seen from Fig. 3 that those solutions found for same weighting factors in different optimization runs are pretty much grouped on the graph and they converge to one part of the Pareto front. It is observed that some solutions on the Pareto front which would make the curve smoother were not found in the given number of iterations. A better Pareto front approximation could certainly be achieved by additional adjusting of weighting factors and more optimization runs.

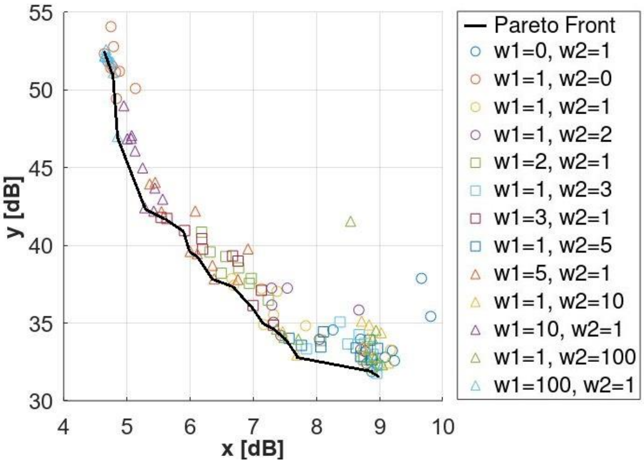

Fig. 3. Pareto Front in terms of root mean square deviations for best-found solutions in frequency range 295 to 305 MHz.

As it is shown in Fig. 2, bandwidth where the front gain is less than 3 dB reduced from the maximum value is about 19 MHz wide. However, in Fig. 3 it is shown that the suppression of the backward lobe has extremely narrow bandwidth, with more than 40 dB differences with frequency shift of 5 MHz, in this case. Therefore, the best-found solutions in terms of average values of gain in the given range are presented in Fig. 4.

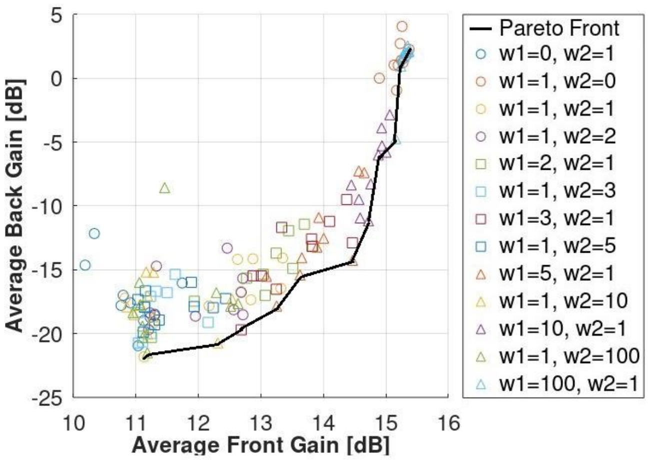

Fig. 4. Pareto Front in terms of average values of gain for best-found solutions in frequency range 295 MHz to 305 MHz.

It can be seen from Fig. 4. that it is possible to keep the average back gain of about -20dBi, up to average front gain of about 12.5 dBi. With further increment of front gain, back gain increases rapidly, entering zone where the objectives start to be more mutually conflicting. Due to the physical nature of the problem, the performances are better in a very narrow bandwidth, as can be seen in Fig. 1 and Fig. 2. Radiation pattern at 300 MHz in H and E plane for one of optimal solutions are given in Fig. 5. and Fig. 6.

V. Conclusion

The best compromises between maximization of gain in the forward direction and minimization of the gain in the backward direction, for twelve-element Yagi antenna in frequency range 295 MHz to 305 MHz are presented and analyzed in this work. It is shown that in order to achieve value of front gain close to maximum possible, one should sacrifice suppression of back lobe level. However, there are numerous optimal solutions for relatively low level of backward radiation, with not so high deviation of front gain from its maximum value. It is also observed that much greater front-to-back ratio could be found in one frequency point, but it is question of significance of these solution when it comes to practical realization and operation of the antenna. Genetic Algorithm used for optimization is confirmed to be suitable for this multi minima optimization problem.

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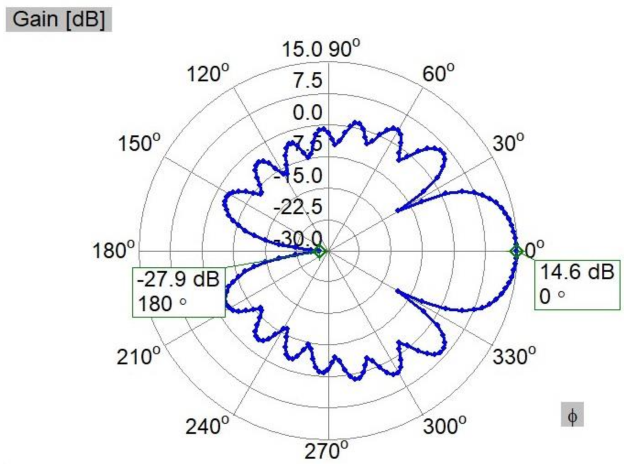

Fig. 5. Gain on 300 MHz in H-plane for one of optimal solutions

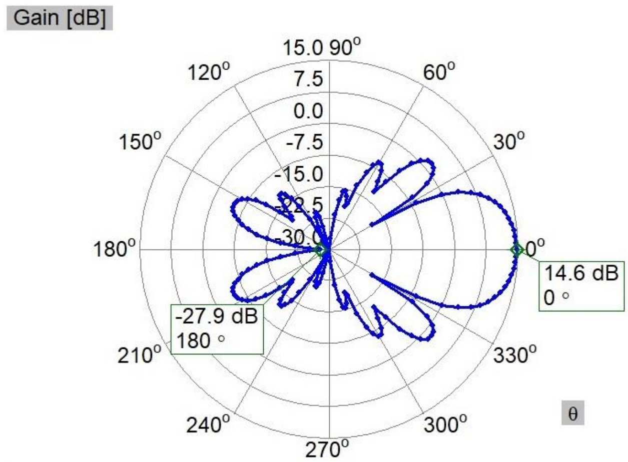

Fig. 6. Gain on 300 MHz in E-plane for one of optimal solution

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