Four-bar Linkage Mechanism Optimization for Linkage Driven Underactuated Robotic Finger

Lazar Matijašević, *PhD Student*, Petar B. Petrović, *Full Professor*

***Abstract* — When designing linkage driven underactuated robotic finger, many parameters needs to be satisfied in order to produce robust robotic hand capable of withstanding industrial environment and capable of fulfilling all needs of robotic assembly in terms of precision and dexterity. For this study, four-bar linkage mechanisms are used to drive underactuated robotic finger and design parameter that will be addressed is transmission performance. Optimization method used for obtaining length of links of four-bar mechanism, based on transmission performance is shown. Freundenstein's analytic method for four-bar linkage function generation, is chosen, and calculated link lengths are to be used for acquiring parameter called transmission defect, parameter that is objective function to be minimized in this optimization process. Maximizing transmission performance, leads to increase of the transmitted torque from the actuated joints to the underactuated joints through transmission mechanism. This paper presents design and kinematic analysis of three degrees of freedom (3-DoF) underactuated robotic finger with linkage driven mechanism for CMSysLab robotic hand.**

***Index Terms*—Robotic Assembly, Robotic Hand, Design; Transmission performance, Transmission defect;**

I. Introduction

Concept of underactuation has many advantages and because of these advantages it is used in many branches of industry. In robotics, it has application in many robotic hands that are designed for laboratory and industrial setting. These hands utilize different mechanisms that transmit the actuation torque to the underactated joints. Two most popular and widely used concepts for underactuated mutlifingered robot hands, [1], are tendon and linkage based transmission mechanisms, shown on Fig.1.

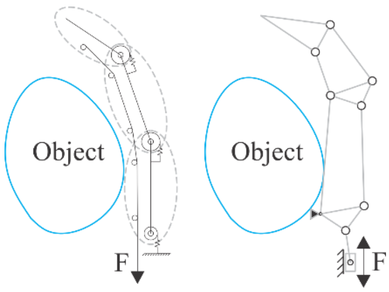

Fig. 1. Tendon and linkage based transmission mechanisms used in underactuated robotic finger. [2].

For our study we have chosen linkage based mechanism because of it's rigidity which makes these mechanisms more predictable, more accurate and more controllable.

PhD student Lazar Matijašević is with the Faculty of Mechanical Engineering, University of Belgrade, Kraljice Marije 16, 11120 Belgrade, Serbia (e-mail: lmatijasevic@mas.bg.ac.rs).

Full Professor Petar B. Petrović is with the Faculty of Mechanical Engineering, University of Belgrade, Kraljice Marije 16, 11120 Belgrade, Serbia (e-mail: pbpetrovic@mas.bg.ac.rs).

These, linkage based mechanisms, are suitable for bigger grasping forces, which is mandatory in assembly operations in industrial setting.

Simplified sketch of underactuated 3-DOF finger with linkage driven mechanism is shown on Fig. 2.

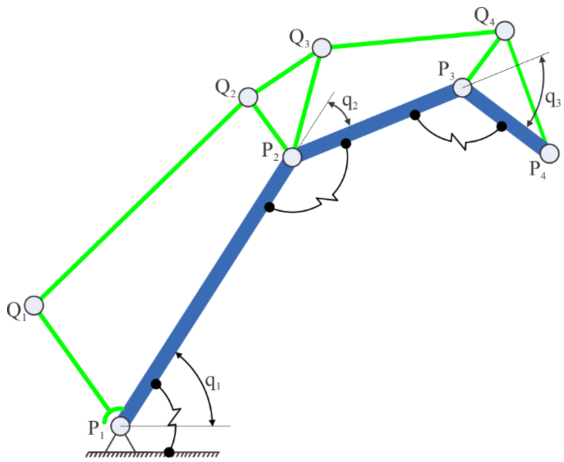

Fig. 2. Representation of 3-DOF finger with linkage driven mechanism.

Transmission mechanism of 3-DOF underactuated finger shown on Fig. 2 consists of two four-linkage mechanisms, O_3-O_2-P_2-P_3 and O_2-O_1-P_1-P_2', interconnected with rigid triangular shaped rocker, O_2- P_2- P_2'.

Requirements for the design of linkage based robotic finger must be met in order to ensure desired motion. Many different optimization algorithms are devised in order to obtain better characteristics of transmission mechanisms. Some of those characteristics are reduced power consumption, reduced structure errors for different mechanisms, force transmission, weight, size and many more. According to aforementioned characteristics there are many design parameters to be met in order to design robust and dexterous robotic hand that can fulfill all requirements of robotic assembly in industrial setting. Focus of this paper will be on transmission performance optimization. Proposed method of optimization focuses on introducing and minimizing parameter named transmission defect and, based on its minimal value, determining optimal value of lengths of four-bar mechanism links.

II. General Concept of Analysis

Variety of useful mechanisms can be formed from a four-link mechanism, shown on Fig. 3, through slight variations, such as changing the character of the pairs, proportions of links, etc. In this paper, two four-bar linkage mechanisms [3], connected with rocker are used for movement and force transmission on phalanx of the underactuated robotic finger.

A. *Freudenstein's equation*

Analytical method of kinematic synthesis for four-bar linkage mechanism used in this paper is Freundenstein's method [4]. Using this method, it is possible to calculate link lengths that accommodate generated function. Also this method is useful in calculating link lengths of four-bar linkage mechanisms because we can use only three positions of mechanism to solve Freundenstein's equation by solving system of three linear equations as will be shown. On Fig 3., a typical four-bar linkage mechanism is shown.

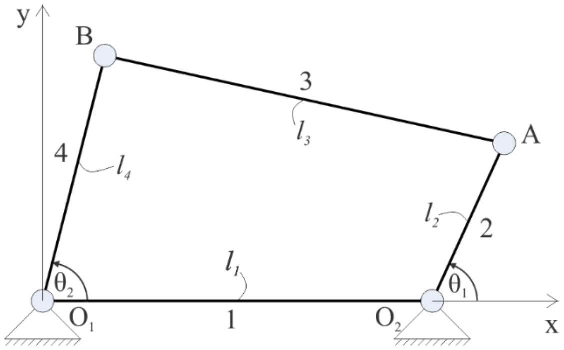

Fig. 3. Representation of four-bar linkage mechanism.

Following parameters are presented on Fig. 3 of O_1O_2AB four-bar linkage mechanism:

- Fixed link O_1O_2, (l_1), also a phalange of robotic finger,
- Input link O_2A, (l_2),
- Coupler link AB, (l_3),
- Output link BO_1, (l_4),
- Orientation of the input and output link is defined by the angles they construct with x axis and those are input angle Θ_1 and output angle Θ_2.

Using three position synthesis method allows us to solve equation by generating two pairs of prescribed coordinated movement, shown in (1), of the input and output link.

$$\left(\theta_1^{12},\theta_2^{12}\right),\left(\theta_1^{23},\theta_2^{23}\right). \tag{1}$$

From position 1 to position 2 change in the angle Θ_1 is given by Θ_1^{12} and corresponding change in the angle Θ_2 is given by Θ_2^{12}. Same stands for movement from position 2 to position 3. These are two pairs of prescribed movement.

In three position synthesis using Freundenstein's method, we assume the values of Θ_1 corresponding to the first configuration (Θ_1^1) and the value of Θ_2 corresponding to first configuration (Θ_2^1). Using inputs from (1) we can write:

$$\begin{aligned}\theta_1^2 &= \theta_1^1 + \theta_1^{12}\,;\\ \theta_2^2 &= \theta_2^1 + \theta_2^{12}\,,\end{aligned} \tag{2}$$

and:

$$\begin{aligned}\theta_1^3 &= \theta_1^2 + \theta_1^{23}\,;\\ \theta_2^3 &= \theta_2^2 + \theta_2^{23}\,.\end{aligned} \tag{3}$$

Freundenstein's equation, which is displacement equation for this planar mechanism, must be valid for all three sets of aforementioned input and output angles. Coordinates of points A and B are:

$$\begin{aligned}x_A &= l_1 + l_2\cos\theta_1\,; \qquad & x_B &= l_4\cos\theta_2\,;\\ y_A &= l_2\sin\theta_1\,, & y_B &= l_4\sin\theta_2\,.\end{aligned} \tag{4}$$

Length of the coupler link 3 is:

$$l_3^2 = (x_A - x_B)^2 + (y_A - y_B)^2\,. \tag{5}$$

Substitution of (4) in (5) yields:

$$\begin{aligned}\cos(\theta_1-\theta_2) = \frac{l_1}{l_4}\cos\theta_1 - \frac{l_1}{l_2}\cos\theta_2 +\\ + \frac{l_1^2+l_2^2-l_3^2+l_4^2}{2l_2l_4}\,.\end{aligned} \tag{6}$$

We now introduce term design parameter D_i, $i=1,2,3$, described as follows:

$$D_1 = \frac{l_1}{l_4};\ D_2 = \frac{l_1}{l_2};\ D_3 = \frac{l_1^2+l_2^2-l_3^2+l_4^2}{2l_2l_4}, \tag{7}$$

and substitute it in (6), which in return leads to Freundenstein's equation for four-bar linkage mechanism shown on Fig. 3:

$$\cos(\theta_1-\theta_2) = D_1\cos\theta_1 - D_2\cos\theta_2 + D_3\,. \tag{8}$$

General form of Freundenstein's equation is:

$$\begin{aligned}\cos\left(\theta_1^i-\theta_2^i\right) = D_1\cos\theta_1^i - D_2\cos\theta_2^i + D_3\,;\\ i = 1,2,3\,.\end{aligned} \tag{9}$$

From there we can solve three linear equations using angle values determined in (2) and (3) and calculate design parameters D_i, $i=1,2,3$. These design parameters are used to calculate length values l_2, l_3, and l_4. Length of link l_1 is chosen for example, based on characteristics of an index human finger or any other assumption. These ratios are important because they allow us to scale robotic finger but to keep same relative movement between various links.

B. Transmission angle

Now, concept of transmission angle [5], needs to be introduced as well, because that is another parameter, besides aforementioned design parameters D_i, $i=1, 2, 3$, that are used in optimization method described in this paper. Let's assume that input torque T_1 is acting on link O_2A. That link in now called input link.

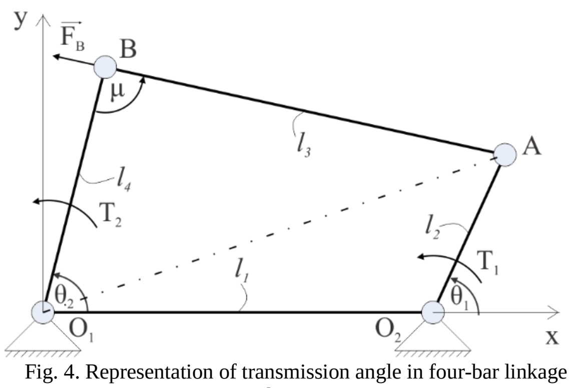

Fig. 4. Representation of transmission angle in four-bar linkage mechanism.

In Fig. 4, if O_2A is the input link, torque T_1 to the input link is causing force $\overrightarrow{F_B}$, which is transmitted through the coupler link *AB*. For sufficiently slow motions (where we can neglect inertia forces), the force in the coupler link is pure tension or compression and is directed along *AB*. For a given force in the coupler link, the torque transmitted to the output link, T_2 (about point O_1), is maximum when the angle μ between coupler link *AB* and output link O_1B is 90º. Therefore, angle ABO_1 is called **transmission angle.**

It varies throughout the range of operation from acute angle μ to obtuse angle π-μ.

According to Fig. 4 transmission angle can be calculated by writing the cosine theorem for O_1A using the triangles O_1AB and O_1AO_2 and equating the length O_1A. It is expressed in following form:

$$\mu = cos^{-1}\frac{l_1^2 + l_2^2 - l_3^2 - l_4^2 - 2l_2l_1\cos(\pi - \theta_1)}{-2l_3l_4}. \qquad (10)$$

Most favorable value of transmission angle is 90º and recommended variations of transmission angle [5] is 90º±50º. When transmission angle is 0º or 180º transmission of motion is impossible. No torque can be realized on output link if transmission angle is 0º and mechanism is at its dead center position.

C. *Transmission quality*

The term transmission index [6] was introduced as a generalization of the concept of transmission angle. It is one of many performance indexes of one mechanism. For planar four-bar linkages, the transmission index is shown to be the sine of the transmission angle. Just like the transmission angle, which varies with the configuration of the linkage, the transmission index is also dependent on configuration. As a global performance parameter that measure the force and motion transmission of the linkage, transmission quality [7] is used, which is defined in terms transmission index. Transmission quality is suitable to evaluate mechanism performance throughout its whole range of motion and is not defined only to a local evaluation at a single configuration.

Transmission quality of four-bar linkage mechanism from Fig. 4, is defined in following equation:

$$z = \sqrt{\frac{1}{\theta_1^1 - \theta_1^3}\int_{\theta_1^1}^{\theta_1^3} sin^2\,\mu d\theta_1}. \qquad (11)$$

The complement [15] of the transmission quality, is defined as:

$$z' = \sqrt{\frac{1}{\theta_1^1 - \theta_1^3}\int_{\theta_1^1}^{\theta_1^3} cos^2\,\mu d\theta_1}, \qquad (12)$$

and is called **transmission defect** which is an objective function to be minimized, and conforms to:

$$z + z' = 1 \;\; ; \;\; 0 < z' < 1\,. \qquad (13)$$

In this paper, link lengths, which yields minimum transmission defect will be defined. Note that this parameter is only one of many parameters that need to be taken into account when designing underactuated robotic finger, and despite the fact that best transmission of force and movement is desired, lengths of links may differ from values calculated here. It is important to keep in mind that these robotic hands should be as light as possible, robust and in order to be industry acceptable they need to be cost efficient.

It is also stated that with mechanisms that have reversal motion, as robotic hand has (opening and closing of hand) transmission angle must be investigated for both cases of motion transmission but that will be scope of future research because in this research only pinch and form grips are taken into account, and in those, direction of forces acting upon object of manipulation are only in direction of closing of robotic hand and that direction will be examined.

In next chapter, analysis described above, will be used on optimization of design of CMSysLab underactuated robotic hand.

III. Synthesis of Optimal Planar Four-Bar Linkage Mechanism for CMSysLab Underactuated Robotic Finger

When designing transmission mechanism, the crucial issue is selection of mechanism and dimensioning of chosen mechanism. Optimization of planar four-bar linkage [8] mechanism is carried out, as mentioned in previous chapter, with regard to three positions of input and output links which yields three linear equations. These are solved using Freudenstein's equation. Later, optimization of force transmission is carried out using transmission defect as objective function to be minimized. The linkages connected in series are synthesized by starting from the four-bar linkage, which moves the distal phalanx.

A. *Synthesis of the four-bar linkage O_3, O_2, P_2, P_3*

For synthesis of the function generating four-bar linkage O_3, O_2, P_2, P_3, shown in Fig. 5, we used method described in section II.

Freudenstein's equation for shown mechanism can be expressed in the form:

$$\cos(\theta_3^i - \theta_4^i) = D_1\cos\theta_3^i - D_2\cos\theta_4^i + D_3\,, \quad i = 1, 2, 3. \qquad (14)$$

Design parameters D_i, $i=1, 2, 3$, based on (7) are calculated as:

$$D_1 = \frac{L_2}{c_2};\; D_2 = \frac{L_2}{a_2};\; D_3 = \frac{L_2^2 + a_2^2 - b_2^2 + c_2^2}{2a_2c_2}. \qquad (15)$$

The four-bar linkage shown in Fig. 5, consists of links a_2, b_2, c_2 of four-bar linkage: O_2P_2, P_2P_3 and P_3O_3 and medial and distal phalanges L_2 and L_3 of underactuated robotic finger. As described in [1], during motion phalanges move until they come in contact with object that needs to be manipulated. Phalanx that comes in contact with object stops moving, in our case medial phalanx L_2, which is fixed on Fig. 5, and distal phalanx L_3 can move until it reaches object or mechanical hardstop which constricts its movement.

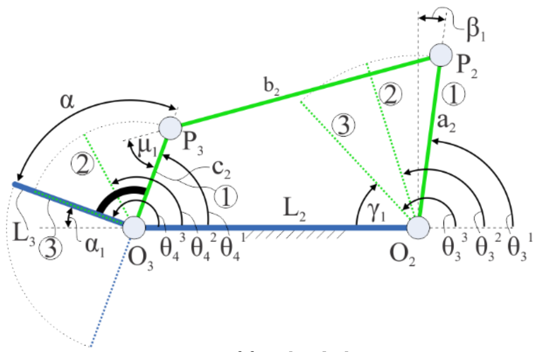

Fig. 5. Representation of four-bar linkage O_3, O_2, P_2, P_3.

Angles θ_3^i and θ_4^i for i=1, 2, 3 are input and output angles of mechanism, respectively.

Many of design parameters on Fig. 5 are empirically acquired, based on actual human hand proportions. According to the proposed mechanical design of the finger, the design parameters are α = 90º, α_1 = 20º, β_1 = 7.5º and γ_1 = 47º. With that in mind, pairs of angles for the starting and final position of links a_1 and c_1 are easily acquired. Values of those angles are (θ_3^1, θ_4^1) = (82.5º, 70º) for starting position and for final position (θ_3^3, θ_4^3) = (133º, 160º).

Freudenstein's equation (14) can be solved when three positions ①, ② and ③ of the input link a_2, and corresponding three positions ①, ② and ③ of the output link c_2 are known. Angle pairs, determined above, are to be substituted in (14), and by solving three linear equations we obtain design parameters D_i, *i=1, 2, 3*, that are used in (15) to calculate link lengths.

Since only two of the three pairs of angles required by the Freudenstein's equations are assigned as design specification of the function generating four-bar linkage *ABCD*, an optimization procedure in terms of force transmission has been developed by varying values of (θ_3^2, θ_4^2) during process of optimization.

These values of link lengths are then substituted in transmission angle equation for four-bar linkage mechanism shown on Fig. 5:

$$\mu_1 = cos^{-1}\frac{L_2^2 + a_2^2 - b_2^2 - c_2^2 - 2L_2 a_2 \cos(\pi - \theta_3)}{2b_2c_2}. \qquad (16)$$

Transmission defect for mechanism on Fig. 7 is defined as:

$$z' = \sqrt{\frac{1}{\theta_3^3 - \theta_3^1}\int_{\theta_3^1}^{\theta_3^3} cos^2\,\mu_1 d\theta_3}\,. \qquad (17)$$

Substituting (16) in (17) gives the value of z'. If it is minimum value, calculated lengths of links will be optimal, if not, then angle values (θ_3^2, θ_4^2) should be varied in a specific range until minimum value of z' is found.

Proposed algorithm

Require: Defining the base phalanx length L_2 and pairs of minimum and maximum values of the input and output angles (θ_3^1, θ_4^1) and (θ_3^3, θ_4^3) of four bar linkage

Step 1: Substitute angle pairs into Freudenstein's equation

Step 2: ***FOR*** (defined increment)
Symbolic solving the system of three equations, resulting in equations for lengths a_2, b_2 and c_2, which are function of angles (θ_3^2, θ_4^2);
end FOR

Step 3: ***FOR*** (defined increment)
$(\theta_3^1, \theta_4^1) < (\theta_3^2, \theta_4^2) < (\theta_3^3, \theta_4^3)$
Solve:

$$\cos(\theta_3^i - \theta_4^i) = D_1 \cos\theta_3^i - D_2 \cos\theta_4^i + D_3\,, i = 1, 2, 3.$$

Calculated values store in predefined matrices A, B and C

Solve:

$$z' = \sqrt{\frac{1}{\theta_3^3 - \theta_3^1}\int_{\theta_3^1}^{\theta_3^3} cos^2\,\mu_1 d\theta_3}\,.$$

Calculated values store in predefined matrix D
end FOR

Step 4: ***WHILE*** z' ~ min (z')
Go through matrices A, B, C and D
end WHILE

Step 5: Obtain values of a_2, b_2 and c_2 for z' = min (z')

Values of link lengths obtained from optimization process are shown in Table III and Fig. 6.

TABLE I
INPUT PARAMETERS FOR OPTIMIZATION ALGORITHM USED FOR OPTIMIZATION OF THE FOUR-BAR LINKAGE O3, O2, P2, P3

Minimum and maximum values of input and output angles			
θ_3^1	82.5º	θ_3^3	133º
θ_4^1	70º	θ_4^3	160º

TABLE II
STARTING VALUES OF PARAMETERS FOR OPTIMIZATION ALGORITHM USED FOR OPTIMIZATION OF THE FOUR-BAR LINKAGE O3, O2, P2, P3

Starting values of parameters to be optimized with limits of values	
θ_3^2	107.75° ± 25°
θ_4^2	115° ± 45°
L_2	37.5 (fixed value)
a_2	23mm ± 10mm
b_2	36mm ± 10mm
c_2	14mm ± 5mm
z'	0.103

TABLE III
OBTAINED PARAMETERS FROM OPTIMIZATION

z'=0.02526			
L_2	37.5	a_2	28.361
b_2	36.993	c_2	15.82

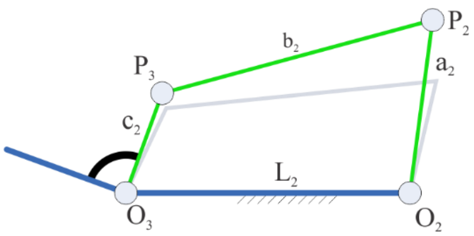

Fig. 6. Optimized parameters of four-bar linkage O_3, O_2, P_2, P_3.

On Fig. 6, four-bar mechanism of grey color is proportional to starting dimensions of link lengths. Green four-bar mechanism is shown with calculated link lengths. It is obvious that there is difference between calculated values after optimization and starting values. It is because starting values are empirically acquired based on human finger proportions and many more parameters like space limitations, etc. All those parameters need to be taken into account when designing underactuated robotic finger and not just transmission quality as described in this paper. It is obvious that this is iterative process and it needs to accommodate many criteria in order to produce optimal result.

Now dimensions of links of second four-bar linkage will be addressed.

B. Synthesis of the four-bar linkage O_2, O_1, P_1, P_2'

After optimization of four-bar transmission mechanism of medial phalanx it is important to do the same with four-bar transmission mechanism of proximal phalanx.

For synthesis of the function generating four-bar linkage O_2, O_1, P_1, P_2', shown in Fig. 7, we used same method as for aforementioned four-bar linkage. Freudenstein's equation for shown mechanism can be expressed in the form:

$$\cos(\theta_1^i - \theta_2^i) = D_1 \cos\theta_1^i - D_2 \cos\theta_2^i + D_3\,, \quad i = 1, 2, 3. \tag{18}$$

Design parameters D_i, $i=1, 2, 3$, based on (7) are calculated as:

$$D_1 = \frac{L_1}{c_1};\ D_2 = \frac{L_1}{a_1};\ D_3 = \frac{L_1^2 + a_1^2 - b_1^2 + c_1^2}{2a_1c_1}. \tag{19}$$

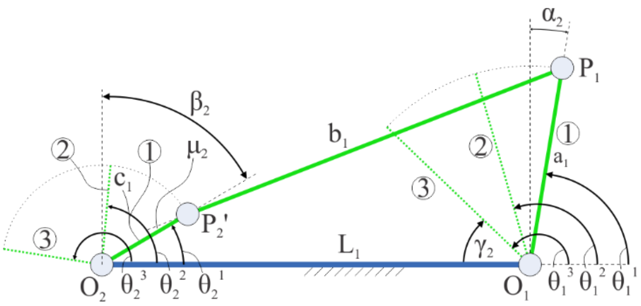

Fig. 7. Representation of four-bar linkage O2, O1, P1, P2'.

Parameters shown on Fig. 7: L_1 is the length of proximal phalanx of the finger, a_1, b_1 and c_1 are lengths of links of four-bar linkage: O_1P_1, P_1P_2' and P_2'O_2, angles θ_1^i and θ_2^i for i=1, 2, 3 are input and output angles of mechanism, respectively.

As already shown, (18) can be solved when three positions ①, ② and ③ of the input link a_1, and corresponding three positions ①, ② and ③ of the output link c_1 are predefined.

Many of design parameters on Fig. 7 are empirically acquired, based on actual human hand proportions. According to the proposed mechanical design of the finger, the design parameters are $\alpha_2 = 10^\circ$, $\beta_2 = 60^\circ$ and $\gamma_2 = 43^\circ$. With that in mind, pairs of angles for the starting and final position of links a_1 and c_1 are easily acquired. Values of those angles are (θ_1^1, θ_2^1) = (80°, 30°) for starting position and for final position (θ_1^3, θ_2^3) = (137°, 171°). These are to be substituted in (18), and by solving three linear equations we obtain design parameters D_i, $i=1, 2, 3$, that are used in (19) to calculate link lengths. These values are then substituted in transmission angle equation:

$$\mu_2 = cos^{-1}\frac{L_1^2 + a_1^2 - b_1^2 - c_1^2 - 2L_1a_1\cos(\pi - \theta_1)}{2b_1c_1}. \tag{20}$$

Transmission defect for mechanism on Fig. 7 is defined as

$$z' = \sqrt{\frac{1}{\theta_1^3 - \theta_1^1}\int_{\theta_1^1}^{\theta_1^3} cos^2\,\mu_2 d\theta_1}\,. \tag{21}$$

Substituting (20) in (21) gives the value of z'. If it is minimum value, calculated lengths of links will be optimal, if not, then angle values (θ_1, θ_2) should be varied in a specific range until minimum value of z' is found, the same way as with previous four-bar linkage. Values of link lengths obtained from optimization process are shown in Table VI and Fig. 8.

TABLE IV
INPUT PARAMETERS FOR OPTIMIZATION ALGORITHM USED FOR OPTIMIZATION OF THE FOUR-BAR LINKAGE O2, O1, P1, P2'

Minimum and maximum values of input and output angles			
θ_1^1	80	θ_1^3	137°
θ_2^1	30°	θ_2^3	171°

TABLE V
STARTING VALUES OF PARAMETERS FOR OPTIMIZATION ALGORITHM USED FOR OPTIMIZATION OF THE FOUR-BAR LINKAGE O2, O1, P1, P2'

Starting values of parameters to be optimized with limits of values	
θ_1^2	108.5° ± 28°
θ_2^2	100.5° ± 70.5°
L_1	37.5 (fixed value)
a_1	32mm ± 10mm
b_1	60mm ± 10mm
c_1	15mm ± 5mm
z'	0.081

TABLE VI
OBTAINED PARAMETERS FROM OPTIMIZATION

z'=0.02993			
L_1	64.5	a_1	41.83
b_1	60.554	c_1	16.39

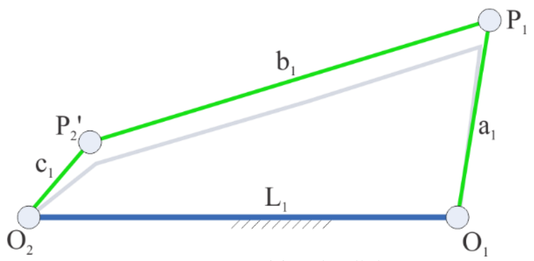

Fig. 8. Optimized parameters of four-bar linkage O2, O1, P1, P2'.

On Fig. 8, with grey color, four-bar mechanism with starting link lengths is shown. Green color represents new four-bar mechanism with calculated link lengths.

Previously described method of optimization of four-bar linkage geometry is useful for process of designing linkage based underactuated robotic hand, because calculated values, now can be used as starting values for later designing processes which will use another set of criteria for acquiring optimal geometry of underactuated finger based on many criteria and not just one described in this paper.

IV. CONCLUSION

In this paper, a method for obtaining optimal lengths of links based on transmission, quality in a four-bar linkage mechanism, was presented. This kind of analysis is important so that researchers and designers of underactuated robotic hands can design proper underactuated finger that can transmit sufficient force onto object that been manipulated with.

As shown, this is only one of many parameters that need to be addressed in order to design proper, robust and dexterous enough robotic hand that is able to accommodate all requirements placed upon it by industrial assembly operations. Some of those requirements are:

Precision: assembly process, especially when we take into consideration precise assemblies in auto or aerospace sectors of manufacturing engineering, require high precision and accuracy of placing of different parts in a assembly.

Strength of grip: it is needed in order for robotic hand to be able to counteract contact forces when handling objects. Rigidity and errors that are imposed from lack of rigidity is important factor to take into account. Errors and lack of rigidity are direct consequence of clearances that exist in assemblies.

Access to the zone of assembly: access to the zone in which assembly of parts needs to take place in required position and orientation.

ACKNOWLEDGMENT

This research work is supported by the Serbian Ministry for Education, Science and Technology Development through the projects titled 'Smart Robotics for Customized Manufacturing', grant No.: TR35007, and 'Development and Experiments of Mobile Collaborative Robot With Dual-Arm', grant No.: 401-00-00589/2018-09, which is jointly realized by Anhui University of Technology, Ma'anshan, Anhui, and Tsinghua University, Beijing, from one side, and Institute Mihajlo Pupin – Belgrade (IMP), Faculty of Mechanical Engineering, University of Belgrade (MFB), and Faculty of technical sciences of Novi Sad, University of Novi Sad (FTN), from the other side, supported by a cluster of industrial partners from both sides, as a bilateral scientific project of high national relevance with the People's Republic of China.

REFERENCES

[1] Birglen, L., et al. "*Underactuated Robotic Hands*", Springer, Berlin, 2008.

[2] Matijasevic, L, Milivojevic, M, Petrovic, P, "Multifingered under-actuated hands in robotic assembly", *13th International Scientific Conference MMA 2018 Flexible Technologies*, Proceedings, ISBN 978-86-6022-094-5, pp.91-94.

[3] L. Birglen ; C.M. Gosselin "Kinetostatic Analysis of Underactuated Fingers", *IEEE Transactions on Robotics and Automation*, vol. 20 Issue: 1, pp.211 -221, april 2004

[4] F. Freudenstein, "Approximate Synthesis of Four-Bar Linkages", *ASME Trans*., Vol.77, No.8, pp.853–861, August 1955.

[5] S. S. Balli ; S. Chand "Transmission angle in mechanisms (Triangle in mech)", in Mechanism and Machine Theory, Elsevier vol. 37 Issue: 2, pp.175-195, 2002.

[6] Chen, C., & Angeles, J.: "Generalized transmission index and transmission quality for spatial linkages*". Mechanism and Machine Theory, 42(9), 1225–1237,* 2007.

[7] Angeles, J., & Bernier, A.:"The Global Least-Square Optimization of Function-Generating Linkages", *Journal of Mechanisms Transmissions and Automation in Design, 109(2), 204,* 1987.

[8] C. Gosselin ; J. Angeles "Optimization of planar and spherical function generators as minimum-defect linkages", in Mechanism and Machine Theory, Elsevier vol. 24 Issue: 4, pp.293-307, 1989.