

GPU-accelerated simulation environment for performance of relay signal adopting DF technique influenced by η - μ fading

Selena Vasić, Nenad Petrović, Stefan Panić, Dejan Milić, Suad Suljović

Abstract — In this paper, the performance of cellular communication system operating over η - μ fading channel, is considered. Using a Decode-and-Forward (DF) multi-hop relay system transmission performances can be enhanced. The closed-form expression for the outage probability (P_{out}) of the system has been derived. In the proposed system, the transmission time has been divided into two phases to be used by the decode and forward protocol. In the first phase, the source transmits its data whereas the relays and destination nodes are in receiving mode. In the second phase relay communicates to destination point by sending the recorded information. Moreover, we introduce a GPU-enabled simulation tool to illustrate how the derived expression within scenarios related to smart city mobile networks. According to the achieved results, leveraging GPU significantly accelerates fading calculation for this kind of analysis.

Index Terms— η - μ fading; Decode-and-Forward system; GPGPU; Cumulative distribution function (CDF); Outage probability (P_{out}); Smart city; Quality of Service (QoS).

I. INTRODUCTION

In cellular communications, on its path between transmitter and receiver, signal undergoes multiple degrading effects (such as fading, shadowing and interferences). Fading is one of the main causes of performance degradation of the receiver. The multipath (short-term) fading can be modeled with several distributions such as: Rayleigh, Rice, Nakagami- m , Weibull, η - μ or k - μ [1]. There are many different techniques engaged in mitigating fading effects. Relay transmission has proven as practical communication technique providing efficient coverage in ad hoc and WLAN communication networks.

The impact of η - μ fading on wireless communication system performances is discussed in several papers. In [2], the first-order statistics for the distribution of η - μ and α - k - μ , eigenvalue functions and cumulative distribution functions are analysed. In [3], the outage probability of selection combining

diversity receiver in the presence of η - μ short-term fading and Gamma long-term fading is evaluated. Effective managing of radio environment assumes correct predictions and thorough description of the system's performance [4]. In order to cope with the increased demands in data rates in wireless networks, development of relay multi-hop transmission has been given significant attention. Multi-hop transmission is a communication technique that offers benefits in a wider coverage without involving high power on the transmitter side. Using spatial/multi-user diversions this approach facilitates communication through ad-hoc networks where nodes are able to communicate without central control. Multi-hop transmission is built on the concept where a mobile terminal transmits the signal between a base station (BS) and neighboring mobile stations (MS). We consider the case of a deep fading channel between the BS and the initial MS. As a result, source signals (S) on their way to destination (D) are transmitted over multiple paths [5].

Based on the complexity of the relay utilized there are two main types of multi-hop transmission systems: amplify-and-forward (non-regenerative) or decode-and-forward (regenerative) systems [6]. The first uses relays of a simpler design that are only able to amplify and forward the incoming signal without decoding it. They technically function as analog repeaters. On the other side, DF technique engages more complex relay that decodes the signal obtained from the source and then re-encodes it before the signal is sent to the destination. It regenerates the original information from the previous node before it retransmits the information to the next node [7]. In harshly affected channels, the relay takes more time to decode the received signal leading to a lack of efficiency and poor performance of the wireless telecommunication system [8]. The power of the transmitted signal at the relay can dominate the power of the useful signal at the relay. The strength of the emitted signal transmitted by relay using DF technique depends on the relay power constant c , while for $c > 1$ the signal is re-emitted with less power [9].

In this paper we consider a cellular communication system operating over η - μ fading channel. One of the statistical characteristics needed to evaluate the performance of the system operating in a fading environment is the system outage probability (P_{out}). It is needed in order to meet the requirements of Quality of Service (QoS). Once P_{out} is evaluated, it can be used in further calculations of the coverage of the observed cellular system. Within the coverage

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area of the cellular system P_{out} is less than the predefined threshold. Also, it allows for evaluation of the minimum distance between two base stations such that a frequency area can be reused. The minimum distance between two base stations is co-channel interference (CCI) reduction factor and is used in the spectral analysis of the system [10]. It is also important to determine the model to select the area for frequency reuse. The outage probability can be obtained from the cumulative distribution function (CDF) of power $\eta\text{-}\mu$ random variables. Analytical and numerical results of this research can be used to design the optimal receiver for the cellular system operating over $\eta\text{-}\mu$ multipath fading channel.

II. MODEL OF SYSTEM

A model of regenerative relay system using DF technique is presented in Fig. 1. The system consists of source (S), relay (R) and receivers (D). Source-Relay link (S-R) is affected by $\eta_1\text{-}\mu_1$ fading with the power γ_1 . Link Relay- Destination (R-D) undergoes the influence of $\eta_2\text{-}\mu_2$ fading with the power γ_2 .

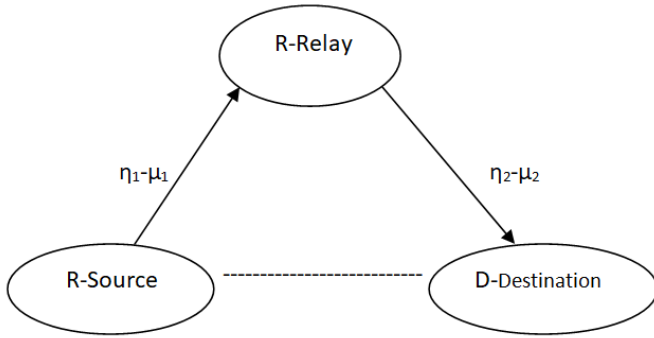


Fig. 1. Model of DF system under the influence of $\eta\text{-}\mu$ fading.

Multi-hop relaying technology is an effective solution in cellular and ad-hoc wireless communications systems. When engaging DF relay signal transmission, each transmission cycle has two stages: the one when the source S communicates with the relay and the destination point and the second when the relay decodes the signal from the source, re-encodes it and sends to the destination D.

In this paper, the value of the power constant of relay c is set to one. Using Mathematica and Origin software package the outage probability P_{out} of system will be first analytically derived and then presented graphically. The first order statistical parameters such as probability density function (PDF) and cumulative distribution function (CDF) of the signal affected by will be calculated. P_{out} is defined as the probability that the value of the SNR (signal-to-noise ratio) at reception is below the value of a predefined threshold (γ_{th}). Based on the obtained first- and second-order statistical characteristics at reception, analysis and performance evaluation of the entire wireless system without adoption of diversion techniques can be conducted [7].

The PDF of the signal x envelop modeled by $\eta\text{-}\mu$ fading, on either of the two links (S-R or R-D) can be written as [11]:

$$p_x(x) = \frac{4\sqrt{\pi}h^\mu x^{2\mu}}{\Gamma(\mu)H^{\mu-1/2}} \left(\frac{\mu}{\Omega}\right)^{\mu+1/2} e^{-\frac{2\mu h x^2}{\Omega}} I_{\mu-1/2}\left(\frac{2\mu H x^2}{\Omega}\right) \quad (1)$$

Here, x represents the instantaneous SNR, Ω is average SNR, μ is the number of clusters, H is the parameter of the phase component, and h denotes fading parameter that describes the powers in phase. Furthermore, $I_\nu(\cdot)$ stands for modified Bessel function of the first kind, while $\Gamma(\cdot)$ stands for Gamma function. Variances of independent Gaussian processes in phase and quadrature, are random and in ratio defined by the following parameter: $\eta = E(X_{I_r}^2)/E(X_{Q_r}^2)$.

Using the expression for modified Bessel function of the first kind: [12; 8.445]:

$$I_\nu(x) = \sum_{k=0}^{\infty} \frac{(x/2)^{\nu+2k}}{k! \Gamma(\nu+k+1)} \quad (2)$$

the expression (1) for signal's PDF, under the influence of $\eta\text{-}\mu$ fading becomes:

$$p_x(x) = \frac{4\sqrt{\pi}h^\mu}{\Gamma(\mu)} e^{-\frac{2\mu h x^2}{\Omega}} \sum_{i=0}^{+\infty} \frac{H^{2i} x^{4i+4\mu-1}}{\Gamma(i+\mu+1/2)i!} \left(\frac{\mu}{\Omega}\right)^{2i+2\mu} \quad (3)$$

By introducing the new parameter γ that describes the conversion of the signal strength to signal-to-noise ratio (SNR) we obtain the expression for the square of the instantaneous signal-to-noise value [13; 2.3]:

$$x^2 = \frac{\gamma}{w} \Omega, |J| = \frac{dx}{d\gamma} = \frac{1}{2\sqrt{\gamma}} \sqrt{\frac{\Omega}{w}} \quad (4)$$

In the above equation, w denotes the average SNR and J holds for Jacobian of the random variable transformation. Hence, the PDF of the instantaneous signal [13; 2.3] is:

$$p_\gamma(\gamma) = p_x(x)|J| = \frac{2\sqrt{\pi}h^\mu}{\Gamma(\mu)} e^{-\frac{2\mu h \gamma}{\bar{\gamma}}} \sum_{i=0}^{+\infty} \frac{H^{2i} \gamma^{2i+2\mu-1}}{\Gamma(i+\mu+1/2)i!} \left(\frac{\mu}{\bar{\gamma}}\right)^{2i+2\mu} \quad (5)$$

Furthermore, the cumulative distribution function (CDF) of the instantaneous SNR value [11] will be:

$$F_\gamma(\gamma) = \int_0^\gamma p(t) dt = \frac{\sqrt{\pi}}{\Gamma(\mu)} \sum_{i=0}^{\infty} \frac{H^{2i}}{2^{2i+2\mu-1} h^{2i+\mu}} \cdot \frac{1}{i! \Gamma(i+\mu+1/2)} \gamma \left(2i+2\mu, \frac{2\mu h}{\bar{\gamma}} \gamma\right) \quad (6)$$

In the expression (6) γ (b, c) denotes a lower Gamma function. It can be represented by lower or complementary incomplete Gamma function $\Gamma(\alpha, x)$, and Gamma function $\Gamma(\cdot)$ [12; 8.356.3] as follows:

$$\gamma(n, x) = \Gamma(n) \left(1 - e^{-x} \sum_{k=0}^{n-1} \frac{x^k}{k!} \right) \quad (7)$$

Combining the last two expressions, the following form of the CDF of the SNR with η - μ distribution becomes:

$$F_\gamma(\gamma) = \frac{\sqrt{\pi}}{\Gamma(\mu)} \sum_{i_1=0}^{\infty} \frac{H^{2i_1} \Gamma(2i_1 + 2\mu)}{2^{2i_1+2\mu-1} h^{2i_1+\mu} i_1! \Gamma(i_1 + \mu + 1/2)} \cdot \left(1 - e^{-\frac{2\mu h}{\bar{\gamma}} \gamma^{2i_1+2\mu-1}} \sum_{i_2=0}^{\infty} \frac{1}{i_2!} \left(\frac{2\mu h}{\bar{\gamma}} \gamma \right)^{i_2} \right) \quad (8)$$

The outage probability of the system (P_{out}) for the observed DF relay signal transmission is given by [9; 2.34]:

$$P_{out} = \int_0^{+\infty} P_\mu \left[\gamma_1 \left\langle \frac{\gamma_{th} c}{\gamma_1 - \gamma_{th}} \middle| \gamma_2 \right\rangle p_{\gamma_1}(\gamma_1) d\gamma_1 \right] = \int_0^{+\infty} F_{\eta-\mu} \left(\frac{\gamma_{th} (c + \gamma_2)}{\gamma_2} \right) p_{\eta-\mu}(\gamma_2) d\gamma_2 \quad (9)$$

In the above formula, the power constant of the DF relay is denoted as c . Combining the expressions (5) and (8) with the expression (9), we can obtain the expression for the P_{out} of a system at the receiver side:

$$P_{out} = \frac{\pi}{\Gamma^2(\mu)} \sum_{i_1=0}^{+\infty} \sum_{i_3=0}^{+\infty} \frac{H^{2i_1+2i_3} \Gamma(2i_1 + 2\mu)}{2^{2i_1+2\mu-2} \Gamma(i_1 + \mu + 1/2) \Gamma(i_3 + \mu + 1/2)} \cdot \frac{(\mu/\bar{\gamma})^{2i_3+2\mu}}{h^{2i_1} i_1! i_3!} \left[\int_0^{+\infty} \gamma_2^{2i_3+2\mu-1} e^{-\frac{2\mu h}{\bar{\gamma}} \gamma_2} d\gamma_2 - e^{-\frac{2\mu h \gamma_{th}}{\bar{\gamma}}} \sum_{i_2=0}^{+\infty} \frac{1}{i_2!} \cdot \left(\frac{2\mu h \gamma_{th}}{\bar{\gamma}} \right)^{i_2} \int_0^{+\infty} \gamma_2^{2i_3-i_2+2\mu-1} (c + \gamma_2)^{i_2} e^{-\frac{2\mu h}{\bar{\gamma}} \gamma_2} e^{-\frac{2\mu h \gamma_{th}}{\bar{\gamma}}} d\gamma_2 \right] \quad (10)$$

In Section III, Fig. 2 depicts the P_{out} on the receiver side (destination D), obtained from the expression (10). P_{out} is given in terms of the mean SNR, denoted by w . The parameters η and μ have been varied in order to observe their effect on the P_{out} .

III. NUMERICAL AND GRAPHICAL RESULTS

In order to observe the effect of fading severity, analytical results are presented graphically in the Fig. 2. The graph of the system P_{out} is obtained under the following conditions on the receiver side: $\mu_1 = \mu_2 = \mu$, $\eta_1 = \eta_2 = \eta$, $\gamma_{th1} = \gamma_{th2} = \gamma_{th}$, $\bar{\gamma}_1 = \bar{\gamma}_2 = \bar{\gamma} = w$.

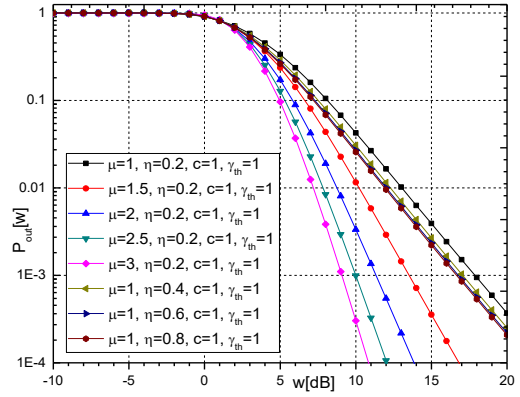


Fig. 2. P_{out} of system, for different values of parameters η and μ .

The outage probability of the system is given by the expression (10). With the increase of parameters μ and η , the P_{out} of system decreases for positive values of the mean SNR, (w in dB) and the system tends to increase its stability. Using program Mathematica, it is calculated that, for example, at $w=5$ dB, for the following values of parameters: $\mu=2$ ($c=\gamma_{th}=1$, $\eta=0.2$) the outage probability is: $P_{out}=17,22\%$, while for $\eta=0.6$ ($\mu=c=\gamma_{th}=1$,) it increases to $P_{out}=27.00\%$. Due to fact that outage probability can be calculated only with predefined numerical error (due to double infinite sum), around 40 summands were required to achieve precise calculation.

IV. SOFTWARE SIMULATION ENVIRONMENT

In this paper, we adopt the derived expression within a software environment which enables modeling and simulation of scenarios related to mobile networks in context of smart cities. The tool is run in web browser and developed relying on JavaScript¹ and HTML² on the front end, while Node.js³ is used for the back end. The implementation of these tools is built upon the previous work regarding the simulation of various aspects related to energy efficiency [14], autonomous vehicles [15], fog computing [16] and resource planning in smart cities [17]. In addition to previous work, Three.js⁴ is adopted in this paper to enable 3D graphics inside the web browser and aspects of mobile network modeling and simulation introduced. The layout of the web-based tool is displayed in Fig. 3. In given screenshot, the mobile network consumers are autonomous cars, while the antenna represents base station providing the mobile network service.

¹ <https://www.w3schools.com/js/>

² <https://www.w3schools.com/html/>

³ <https://nodejs.org/en/>

⁴ <https://threejs.org/>



Fig. 3. Screenshot of a web-based tool for modeling and simulation of smart city mobile network scenarios.

First, user creates a mobile network model using the graphical 3D environment run in a web browser. The aspects related to network infrastructure, terrain (obstacles), communication channel and service consumers are taken into account [18]. The value of Quality of Service (QoS) parameter is calculated based on P_{out} . The user-defined mobile network model together with QoS values is used as input of linear optimization process with respect to linear program presented in [18]. Finally, linear optimization process outputs the optimal base station configuration which can be further translated to Software Defined Radio (SDR) commands. In Fig. 4, an overview of the proposed modeling and simulation environment for network planning is given. However, in this paper, we focus on GPU-enabled fading calculation.

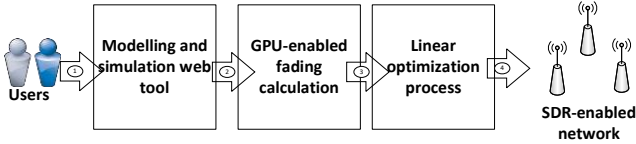


Fig. 4. Overview of software environment: 1-Drawing network model diagram 2-User-defined network model 3-QoS values 4-SDR commands

General-purpose GPU (GPGPU) programming has goal to speed-up the calculations processing huge streams of data in non-graphic applications relying on GPU. It has been approved as effective for fading effect calculation speed improvement [19]. In this paper, NVIDIA CUDA [20] is adopted for GPU-enabled calculation of P_{out} , which is further leveraged for QoS determination for a given location (l) at time point $t+1$, according to the formula:

$$Q_0 S_l(t+1) = Q_0 S_l(t)(1 - P_{out}) \quad (11)$$

The structure of CUDA kernel used for QoS calculation is illustrated as a pseudo-code in Fig. 5.

```

__global__ void qos_factor(Location* l, float* qos_new, float *qos_old)
{
    int tid = threadIdx.x + blockIdx.x * blockDim.x;
    while (tid < N)
    {
        qos_old[tid] = (1-Pout(l[tid])*qos_old[tid];
        tid += blockDim.x * gridDim.x;
    }
}

```

Fig. 5. Pseudo-code of CUDA kernel for QoS factor determination.

According to the achieved results, the GPU-enabled calculation of QoS based on P_{out} , is around 54 times faster than equivalent task done on CPU using Origin and 28 times than our previous work from [21].

Finally, the linear optimization model's objective function [19] aims to minimize the energy consumption, distribution cost and quality of service for base stations, given as:

$$\underset{i \in BaseStation, j \in Location}{\text{minimize}} \sum x[i, j] e_cost[i] d_cost[i, j] qos_drop[i, j] \quad (12)$$

In expression (12), $x[i, j]$ is a decision variable, $e_cost[i]$ is energy consumption of a given $BaseStation[i]$, $d_cost[i, j]$ represents energy distribution cost at $Location[j]$, while $qos_drop[i, j]$ is a ratio between the maximum QoS parameter value and the estimate based on fading calculation for $BaseStation[i]$ at $Location[j]$.

V. CONCLUSION

In this paper, the performance of cellular communication system operating over η - μ fading channel has been considered. η - μ fading model contemplates a general non-line-of sight (NLOS) propagation scenario. By setting two shape parameters η and μ , this model includes some classical fading distributions as particular cases, e.g. Nakagami- q (Hoyt), One-Sided Gaussian, Rayleigh and Nakagami- m . The outage probability (P_{out}) exemplifies a key performance in cellular communications under fading effects. Considering the relevancy of η - μ fading channels, the obtained analytical expression for the P_{out} is of significant interest in analysis of channel performances. We have analyzed the transmission that engages the regenerative Decode-and-Forward (DF) relaying in a η - μ fading environment, without applying diversion techniques. Specifically, the outage probability P_{out} of the cellular relay telecommunication system has been considered. It is observed that an increase in parameter μ , (i.e. a decrease in the number of scattering clusters), leads to improved system performance. With increase of parameter μ , the probability density function becomes more narrow and tends toward a delta function at the position of the mean strength of the signal.

Furthermore, when μ tends to infinity, no-fading scenario is considered. In the observed relay transmission without combining, the outage probability depends mostly on the receiver sensitivity threshold. We have set the parameter c , that describes the amplification (the output power of the DF relay) to one. In addition, an illustration of the real-life scenario is achieved through introducing a web browser tool

for modeling and simulation of smart cities environment. The adoption of GPGPU significantly speeds up the fading effect calculation. To the best of authors' knowledge, similar analysis has not been presented in literature which signifies the contribution of this paper.

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