Utilization of Characteristic Mode Analysis in Coupled Resonators Microstrip Filter Design

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***Abstract*—Examination of resonant frequency and coupling coefficient is essential part in microwave filters design with coupled resonators. We introduce Characteristic Mode Analysis (CMA) based method for calculation of coupling curve, applied to microstrip resonators. The main advantage of this approach is simplicity, due to CMs independency of any external sources. The results for coupling curve are presented and cross-checked with results obtained by equivalent two ports microstrip model with feeding lines. A very good agreement between the two methods is observed.**

***Index Terms*—Coupling coefficient, coupled resonators, CMA, filter, resonant frequency.**

I. Introduction

DISHAL acknowledged that any narrow-band bandpass filter can be described by tuning frequency of the resonators, the couplings between adjacent resonators and the external *Q*-factor of the first and last resonators [1], [2]. Initially, in case of distributed resonators, these parameters had usually been determined without a straight-forward method, but rather with a set of numerous experiments and measurements. With expansion of electromagnetic (EM) solvers, generating design curves for coupling and external *Q*-factor in terms of variables of interest became significantly easier. In [3], it was demonstrated how to extract the curves from insertion loss response and time delay at resonances, obtained from EM simulations. That approach we will later use in order to verify our results.

Since it was introduced, Characteristic Mode Analysis (CMA) was mainly used to analyze radiating properties of various antennas and scattering objects [4]–[6]. Apart from these fields, it found a role in analysis of surface wave resonance as well [7]. In this paper we outline a method for investigating resonant frequencies and generating coupling curve between resonators using CMA theory, in example of microstrip coupled resonators. We haven't yet considered a way to determine external *Q*-factor using CMA, which would allow to apply Dishal's concept [2] with all three variables required for bandpass filter design obtained by CMA only. The main purpose of this paper is to indicate a potential of CMA as a tool in narrow-band microwave bandpass filter design.

Chosen example for presenting the method consists of two grounded $\lambda/4$ resonators in microstrip technique, designed on operating frequency 1 GHz. To take advantage of Characteristic Modes formulation for PEC bodies [8], we considered air substrate. All the full-wave 3D EM analysis was performed in WIPL-D software package [9], which utilizes MoM (*Method of Moments*) and HOBFs (*Higher Order Basis Functions*).

II. Theoretical Insight to Characteristic Modes Analysis

Characteristic mode analysis is the numerical calculation of a weighted set of orthogonal current modes that are supported on a given structure. The theory was first introduced to electromagnetic by Garbacz [3], and later Harrington and Mautz formulated generalized eigenvalue equation for conducting bodies applied on MoM impedance matrix [8]. Here follows brief insight to derivation of eigenvalue equation. When an incident plane wave $\mathbf{E}^{\mathrm{i}}$ illuminates PEC structure, it induces surface currents $\mathbf{J}_{\mathrm{s}}$, which induce scattered field $\mathbf{E}^{\mathrm{s}}$. Boundary condition for $\mathbf{E}$ filed on the PEC body surface S can be written as:

$$(\mathbf{E}^{\mathrm{i}}(\mathbf{r})+\mathbf{E}^{\mathrm{s}}(\mathbf{r}))_{\tan}=0,\quad \mathbf{r}\in S, \tag{1}$$

where "tan" denotes tangential components of electric field. Scattering field $\mathbf{E}^{\mathrm{s}}$ can be expressed in terms of induced surface current as:

$$\begin{aligned}\mathbf{E}^{\mathrm{s}}=&-\frac{j\omega\mu_0}{4\pi}\int_S G(\mathbf{r},\mathbf{r}')\mathbf{J}_s(\mathbf{r}')dS'\\&-\frac{j}{4\pi\varepsilon_0\omega}\nabla\int_S G(\mathbf{r},\mathbf{r}')\nabla'\mathbf{J}_s(\mathbf{r}')dS'\end{aligned} \tag{2}$$

where ε_0 and μ_0 are the permittivity and permeability of the free space, and $G(\mathbf{r},\mathbf{r}')$ is Green's function in free space multiplied by 4π, and given by

$$G(\mathbf{r},\mathbf{r}')=\frac{e^{-jk_0R}}{R},\quad R=|\mathbf{R}|,\quad \mathbf{R}=\mathbf{r}-\mathbf{r}', \tag{3}$$

where R is distance between the field and the source point and $k_0=\sqrt{\varepsilon_0\mu_0}$ is the wavenumber in free space. Relationship

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between scattering field and surface currents can be written in form of integro-differential operator $L(\cdot)$, thus boundary condition (1) can be expressed as:

$$\left[L(\mathbf{J}_s)\right]_{\tan} = \mathbf{E}^{\mathrm{i}}_{\tan}(\mathbf{r}),\ \mathbf{r} \in S \tag{4}$$

that is known as EFIE (*Electric Field Integral Equation*). If we write tangential component of $L(\cdot)$ operator as a new operator $Z(\cdot)$, we obtain:

$$\left[L(\mathbf{J}_s)\right]_{\tan} = Z(\mathbf{J}_s). \tag{5}$$

Operator $Z(\cdot)$ has impedance property and can be split into real and imaginary part as:

$$\mathbf{Z} = \mathbf{R} + j\mathbf{X} \tag{6}$$

which represents MoM impedance matrix. By using impedance matrix in weighted eigenvalue equation, generalized eigenvalue equation for CM calculation is defined as:

$$\mathbf{X}(\mathbf{J}_{s,n}) = \lambda_n \mathbf{R}(\mathbf{J}_{s,n}). \tag{7}$$

Solutions of (7) are eigenvectors $\mathbf{J}_{s,n}$, that are vectors of current coefficients, eigenvalues λ_n, and n is the order of each mode. Eigenvalue is the real number within a range $\left[-\infty, +\infty\right]$, and its magnitude is proportional to the total stored field energy:

$$\omega \iiint_V (\mu \mathbf{H}_n \cdot \mathbf{H}_n^* - \varepsilon \mathbf{E}_n \cdot \mathbf{E}_n^*) \mathrm{d}V = \lambda_n. \tag{8}$$

Physical meaning of eigenvalues can be interpreted as follows:

- In the case of $\lambda_n = 0$, stored electric and magnetic energies are equal, and associated modes are considered as the resonant modes.
- In the case of $\lambda_n < 0$, stored electric energy dominates, and associated modes are considered as the capacitive modes.
- In the case of $\lambda_n > 0$, stored magnetic energy dominates, and associated modes are considered as the inductive modes.

The more convenient way for graphical representing eigenvalue is parameter called Modal Significance, defined as:

$$\mathrm{MS} = \left| \frac{1}{1 + j\lambda_n} \right| \tag{9}$$

MS takes values from [0,1], and for resonant modes, when λ_n approaches to 0, it is close to 1.

The important property which can be noticed from (7), is that CMs do not depend on any external excitation, but on the physical properties of the structure only. MoM matrix is filled-in, after which eigenvalue equation is solved in order to calculate unknown current coefficients for each mode, i.e. eigenvectors, as well as eigenvalues. Consequently, there is no need for any feeding network in analysis of the resonant properties in this way.

As it is mentioned before, the reason for analysis of PEC body in free space is to present the research using available tool [9]. In [10], theory of characteristic modes for material bodies is introduced, where the main difference from the theory for perfectly conducting bodies lies in the computation of the modes. It is also discussed in [10] that characteristic modes in material bodies have the most properties as corresponding modes in perfectly conducting bodies. Having that in mind, we may assume that the method is also valid for microstrip with other dielectric properties.

III. Generating Coupling Curve

Generally speaking, coupling coefficient can be defined as a ratio of coupled energy to stored energy, while corresponding electric and magnetic fields should be calculated at resonant frequencies [11]. In [12] it is given the expression for coupling coefficient, derived from lumped-element circuits:

$$k = \left(f_2^2 - f_1^2\right) \Big/ \left(f_1^2 + f_2^2\right), \tag{10}$$

where f_1 and f_2 are the lower and upper resonant frequency of the coupled resonators. Equation (10) is also valid for the distributed resonators, and resonant frequencies can be determined from full wave EM simulations. In following examples, coupling coefficient is always calculated using (10), while required resonant frequencies are obtained from both CMA and insertion loss response from model with feeding lines and ports. As an electromagnetic coupling increases when the elements are getting closer to each other, it is significant to graphically represent it as a function of distance, thus to generate coupling curve.

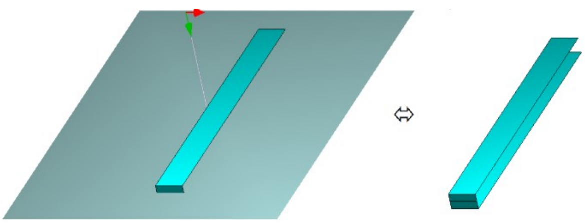

Fig. 1. Single grounded resonator above infinite PEC plane.

A. CMA Based Method and Numerical Model

Firstly, a single λ/4 resonator at operating frequency 1 GHz, with a grounded end above PEC plane, is analyzed with CMA solver. Distance from PEC plane is $h = 2$ mm, length of resonator $l = 74.95$ mm, and width $w = 9.83$ mm. In CMA model it is not possible to define a port between ground and resonators that would introduce a voltage necessary for transmission line. This obstacle is overcome by defining an infinite PEC plane that indicates the image theory to be applied. The original and the equivalent model after image theory are applied as given in Fig. 1.

Analysis is performed in discrete frequency points, in range from 0.8 GHz to 5 GHz. MS for the first 3 modes is shown in Fig. 2.

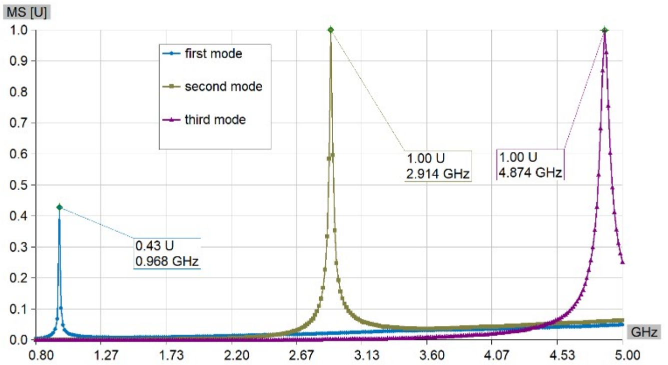

Fig. 2. MS for the first three modes of single resonator in frequency range 0.8 GHz – 5 GHz.

It can be seen from Fig. 2, that in the analyzed range there are three very narrow resonant modes, and the one of interest is at around 0.968 GHz. In order to analyze the coupling, one more resonator is added with the same dimensions, but grounded on different end, as shown in Fig. 3, and the results are given on Fig. 4. This model of coupled resonators is important for interdigital bandpass filter design.

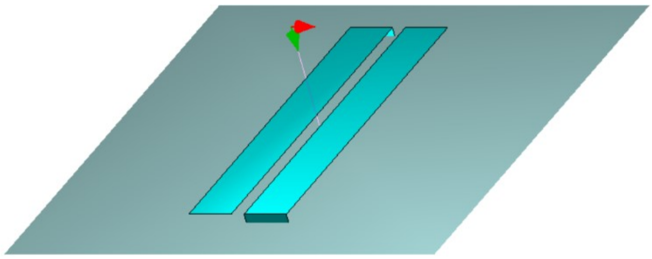

Fig. 3. Two coupled grounded resonators above infinite PEC plane.

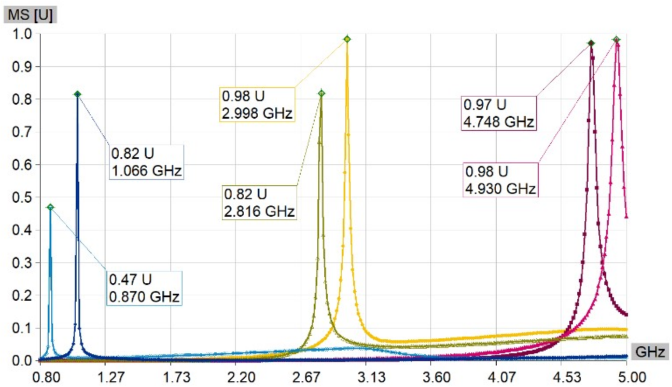

Fig. 4. MS for the first six modes of two coupled resonators in frequency range 0.8 GHz – 5 GHz.

As it was expected, instead of each resonance in case of single resonator, now we have two resonances shifted in frequency. Each peak still comes from different characteristic mode, but the two in every pair is very physically similar to each other, meaning have similar current and near field distribution. With varying the spacing between resonators, the spikes appear in the different positions in frequency. Having the low and high resonant frequencies as the only unknown quantities in (10), coupling coefficient can be calculated easily. In this example for spacing $s = 1$ mm, coupling coefficient equals to 0.2.

B. *Method Based on Insertion Loss Response*

In order to cross-check calculated results for coupling obtained by CMA method, we created the equivalent microstrip model with two resonators and two loosely coupled feed lines, presented in Fig. 5. The dielectric is air. The distances between feed lines and resonators equals to $2h$. Two resonators are grounded and two ports are placed on the different ends of feeding lines. The length and the width of the ground plane equal to 135 mm and 100 mm, respectively. The insertion loss response ($IL = -20\log_{10}|s_{21}|\text{dB}$) is given in Fig. 6, and it can be observed that six resonances appeared at the same frequencies as resonances shown in MS response in Fig. 4.

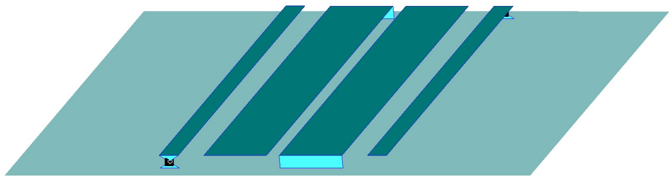

Fig. 5. Equivalent model with resonators and feeding lines.

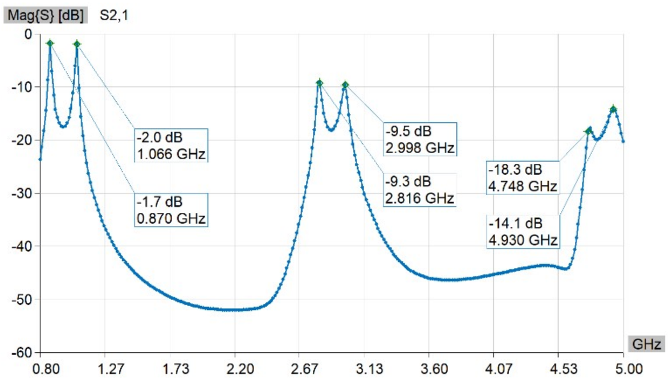

Fig. 6. Insertion loss response in frequency range 0.8 GHz – 5 GHz.

C. *Coupling Curve Results*

In case of both methods, models were re-simulated for different values of spacing between resonators, from range 0.1 mm to 10 mm. For each point, the equation (10) is calculated by inspecting resonances from MS graph and *s*-parameters. Results are overlaid on graph shown in Fig. 7, where "MS" denotes CMA based method, while "s_{21}" denotes the method based on insertion loss response.

The graph from Fig. 7 confirms that these two approaches result in the same coupling coefficient curves, and it can be said the CMA based method is verified. Negligible differences could be reduced by additional increasing of EM simulation accuracy, which is for the purpose of this research considered unnecessary.

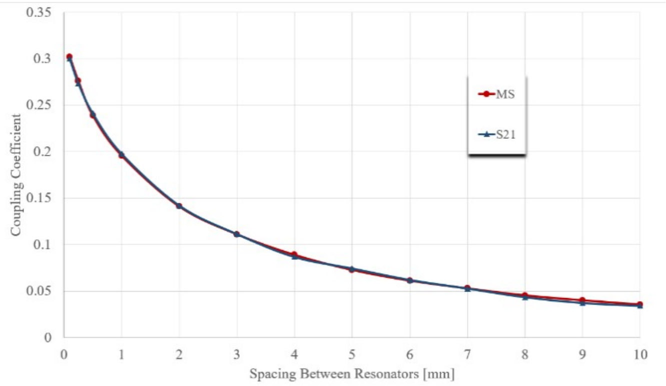

Fig. 7. Coupling curve in terms of spacing between resonators.

IV. Conclusion

In this paper, we have shown how CMA can be used in order to analyze resonant frequencies and coupling between resonators, and therefore can be used in bandpass microstrip filter synthesis. It provides very elegant solution to inspect internal resonances without taking care of feeding network and its potential influence. The validity of the results is confirmed with those obtained by inspecting *s*-parameters of the equivalent model with ports and feed lines.

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