Application of the Angular Dependency of the Zero Moment Point

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***Abstract*— In this paper the widely used stability parameter called the zero moment point (ZMP) is redefined as the angle around the center of mass (COM) of the investigated system. With this redefinition the ZMP is expressed in a more general way which enables its application in a wider range of situations. The angular definition of the ZMP was validated with motion measurements of a person performing different movements recorded with the OptiTrack camera system. The skeleton of the filmed person was reconstructed with the Motive software and a body model was used to reconstruct its COM, which was further used to calculate the ZMP. The stability analysis of the recorded motion measurements presented in this article shows on real-world examples of human motion that the angular redefinition of the ZMP provides a general, reliable and simple-to-apply way of determining the stability of a system.**

I. INTRODUCTION

Since the beginnings of motion control researchers have investigated stability conditions and defined different parameters that reveal whether a systems is stable or not and enable the calculation of possible motions in different situations. The research in the field of robot stability has deepened with the development of robotic systems capable of performing complex motion tasks such as for example humanoid robots performing human-like motion. Achieving stability of humanoid robots can be due to their relatively small feet in comparison to their relatively large body sizes very challenging. This is why also simple tasks such as for example walking, demand for constant stability verification and prediction in order to enable their accomplishment [1], [2], [3], [4]. But as the progress in this field is advancing with an incredible speed, robots became recently also capable of running, jumping and even skiing [5], [6], [7].

A system is capable of performing the desired tasks only if the forces, acting from the support polygon, allow it, which means that they make the system dynamically or statically stable. If the system is supported only by the forces acting from the ground, the support polygon lies within the boundaries of the contact of the system with the ground. In the case of a humanoid supported only by its feet the support polygon extends from its heel to its toes and between the outer edges of its left and its right foot.

One of the parameters defining the system's stability is the zero moment point (ZMP), defined as the location on the ground, where all the forces and torques, acting on the system, can be replaced by only one force [8], [9]. If the ZMP lyes within the support polygon it coincides with the center of pressure and in this case the system is stable. On the other hand, if the ZMP lies outside the support polygon it can not exist as there are no support mechanism outside the support polygon and therefore it is usually refereed to as a fictitious ZMP. In this case the system is not stable and if the system is a humanoid it will flip either over its toes or its heel.

But different systems may have different support mechanism that do not necessarily act on the ground. A humanoid may for example support itself with his arms, that may apply support forces at different heights, or with its bottom, in the case when it is sited. In such scenarios the support polygon does not lie only on the ground but it extends to different heights, as shown in Fig. 1, and therefore the standard

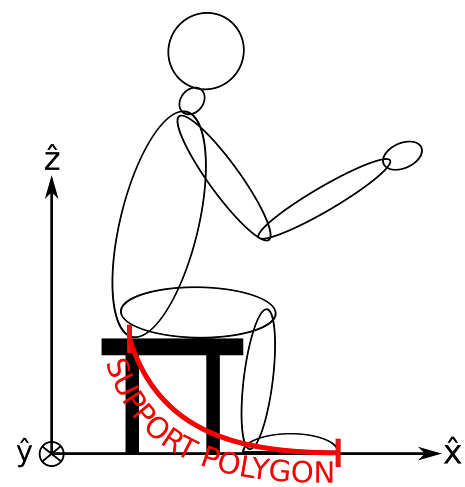

Fig. 1: The support polygon of a humanoid sited on a bench.

definition of the support polygon and the ZMP located on the ground can not be used. This is why in this article the ZMP is expressed in a more general way as the angle around the center of mass (COM) of the investigated system. Such definition can be applied to all systems, also those supported by different mechanisms at different heights, even below the ground or above the system itself.

In this article the stability of a humanoid, which can be either a person or a humanoid robot, is described, but the derived stability conditions can be applied to any system.

II. THE ZERO MOMENT POINT

A. The Standard Cartesian Definition

One of the most widely used stability parameters, the ZMP, can be expressed from the torque balance equation that takes into account the gravitational force acting on each segment of the humanoid, the forces accelerating each segment of the

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humanoid in the desired direction, along with the torques produced by the rotations of each humanoid segment and all the external torques and forces present. By the definition the horizontal torques in the ZMP must be zero and only torques in the vertical direction can exist, but they are in usual circumstances balanced by the friction forces [8], [9].

The coordinate system defining the space in which the humanoid is positioned is oriented such that the sagittal plane of the humanoid lies in the $x-z$ plane, the lateral plane of the humanoid lies in the $y-z$ plane, while the ground lies in the $x-y$ plane, as shown in Fig. 1. This way the x and y components of the torque in the ZMP must be equal to zero, while its z component can be non-zero. For a thorough explanation of the torque balance equation and the derivation of the ZMP see [10].

The torque balance equation is a general equation that takes into account multiple effects that do in numerous circumstances not exist or can be neglected. This is why the position of the ZMP on the ground is usually obtained with some simplified models, one of them being the linear inverted pendulum model [11]. The latter assumes that the investigated system is symmetric with respect to the sagittal plane, it can be approximated by its COM, there are no external forces and torques present, the system does not rotate around any axis and that its COM lies at $y = 0$ and moves only in the x direction. In this case the location on the ground where the ZMP lies can be obtained as

$$x_{\text{zmp}} = x_{\text{com}} + \frac{z_{\text{com}}}{g} \ddot{x}_{\text{com}} \,, \tag{1}$$

where x_{com} and z_{com} are the x nad z coordinates of the COM of the investigated system, respectively, $\ddot{x}_{\text{com}}$ is its acceleration in the x direction, while g is the gravitational acceleration.

But the Cartesian definition of the ZMP may become useless in practice if the ZMP is located too far away from the investigated system, as the system may not have suitable mechanisms to support itself at such distant locations. Furthermore, if the system uses other support mechanisms that are not applying support forces on the ground, which may be in the case of a humanoid its arms and hands, the support polygon does not extend only on the ground and therefore its stability can not be verified using only the ground location of the ZMP. This is why a more general expression of the ZMP was developed.

B. *The Angular Definition*

From the torque balance equation it can be shown that taking into account the same assumptions as for the derivation of the linear inverted pendulum model, with the exceptions that now the ZMP does not need to lie on the ground any more and that the COM of the system can be accelerated also in the z direction, the ZMP can lie anywhere on the ZMP line defined as

$$z_{\text{zmp}} = \tan^{-1}(\varphi_{\text{zmp}})(x_{\text{com}} - x_{\text{zmp}}) + z_{\text{com}} \,, \tag{2}$$

where z_{zmp} and x_{zmp} are the z and x coordinates on the ZMP line, respectively, φ_{zmp} is the ZMP angle defined as

$$\varphi_{\text{zmp}} = -\arctan 2\big(\ddot{x}_{\text{com}}, (\ddot{z}_{\text{com}} - g)\big) \tag{3}$$

and $\ddot{z}_{\text{com}}$ is the vertical acceleration of the COM. φ_{zmp} is located between the vertical line, passing through the COM of the investigated system and the ZMP line, which is in the case, when the assumptions made for the derivation of (2) hold true, passing through the COM of the investigated system, as shown in Fig. 2. With the angular redefinition

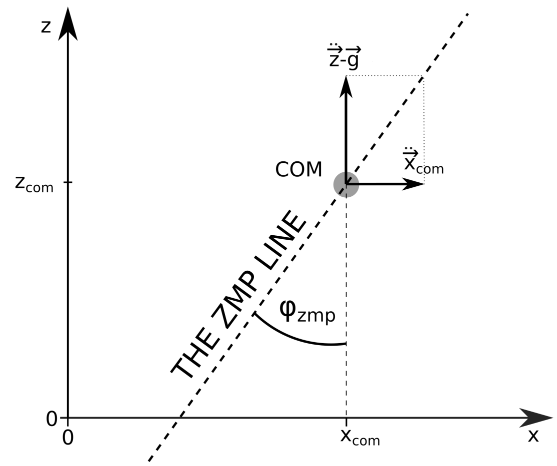

Fig. 2: The angular definition of the ZMP. φ_{zmp} is the angle between the ZMP line and the vertical line, passing through the COM of the investigated system.

the ZMP does not need to lie on the ground any more but it can lie at any height also below the ground, where $z_{\text{zmp}} < 0$, anywhere between the ground and the COM of the investigated system, where $0 \leq z_{\text{zmp}} \leq z_{\text{com}}$, or above the COM, where $z_{\text{zmp}} > z_{\text{com}}$. For a detailed derivation of the angular definition of the ZMP see [10]. To obtain the ZMP angle in the lateral $(y-z)$ plane of the humanoid, the COM coordinates and accelerations in the x direction from (2) and (3) must be substituted with the corresponding values in the y direction.

The stability condition stating that the ZMP must lie within the support polygon can be expressed with the ZMP angle and the angles of the edges of the support polygon in the following way. For the sagittal plane of the humanoid, which is supported by only one foot, or by both feet, positioned in parallel one next to each other, this condition can be expressed as

$$\varphi_{\text{heel}} \leq \varphi_{\text{zmp}} \leq \varphi_{\text{toes}} \,, \tag{4}$$

where

$$\varphi_{\text{heel, toes}} = \arctan\left(\frac{x_{\text{heel, toes}} - x_{\text{com}}}{z_{\text{com}}}\right), \tag{5}$$

as shown in Fig. 3 If the humanoid is supported on the ground with both feet, that are not parallel one to the other, φ_{heel} refers to the heel of the back foot, while φ_{toes} refers to the toes of the front foot. If, on the other hand, the humanoid

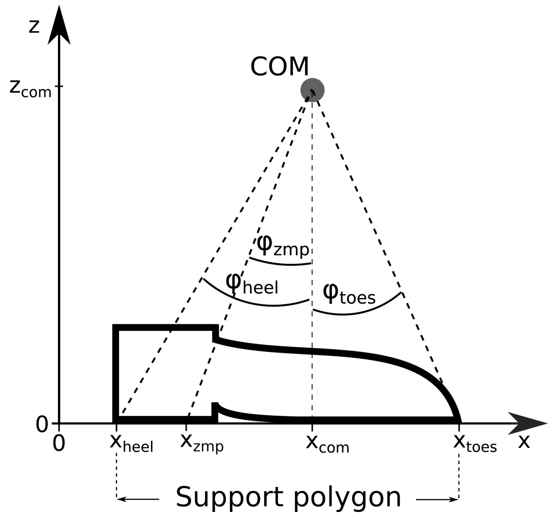

Fig. 3: The angles of the edges of the heel and the toes of the foot of the humanoid with respect to its COM, as defined by (5).

is oriented in the opposite direction, φ_{heel} and φ_{toes} must be interchanged. If the condition (4) does not hold true, the humanoid is not stable and it will flip over its toes or its heel.

III. Validation of the Angular Definition of the ZMP

The angular definition of the ZMP was validated on two measurements of human motion filmed with a set of 13 OptiTrack cameras [12], emitting infrared light and detecting its reflection from 37 reflective markers, positioned on the filmed person. The markers were placed on predefined positions on the human body in order to enable Motive 2.2.0 [13] (the OptiTrack optical motion capture software) to reconstruct the human skeleton. Knowing the approximate positions of the human joints, estimated by the Motive software, and using a human body model [14], [15], [16], which enables the reconstruction of the sizes and masses of each body segment and the distances of their corresponding centers of mass (COMs) from the adjacent joints, the reconstruction of the total COM of the filmed person was possible. The ZMP line and the ZMP angle from (2) and (3), respectively, were then obtained from the reconstructed location and acceleration of the total COM of the person for each recorded frame.

In Fig. 4 you can see the frame sequence outtake from the first filmed motion. The top row shows the lateral plane (front view), while the bottom row shows the sagittal plane (side view) of the filmed person. The frames positioned one above the other were obtained at the same time and therefore represent the same capture. On the first two captures, obtained at times 2.7 s and 4.1 s, the filmed person was swinging from his right side to his left side, on the second two captures, obtained at times 20.6 s and 24.9 s, the person was bowing forth and back, while at the last capture, obtained at time 37.6 s, the person was standing on only one leg, bowing forth, with the other leg lifted up in the air and with his hands extended sideways.

The y and the x coordinates of the ZMP and the edges of the support polygon of the recorded person during the first motion measurement, presented in Fig. 4, where the person was supported only by his feet, are shown in Fig. 5 as a function of time. On the plot on top it can be seen that in the y direction the person was unstable only for small amounts of time during the measurement, when the ZMP is located outside the support polygon. During this measurement the person was not standing on both feet all the time but was also stepping to only one foot while swinging from one side to the other. This can be seen as a sudden narrowing of the support polygon in the y direction, such as for example at time 1.2 s, when the person started to stand only on his left foot, and a sudden widening of the support polygon, when the person was supported again by both feet, such as for example at time 2.0 s. The deviation of the ZMP outside the support polygon occurred during the swinging at around 1.9 s and 15.6 s, when the person was supported only by his left and only by his right foot, respectively, just before he landed to the other foot. This instability arised because just before the landing on the second foot, the person was falling down towards the ground and could not control his motion. Another instability can be seen at time around 14.1 s when the person was standing only on his right foot and was caused by fast and sudden movements of the person while catching balance. As the ZMP was calculated only from the reconstructed position and acceleration of the total COM of the body, the torques produced by the rotations of different body links were not taken into account, which can for fast and sudden movements influence the location of the ZMP. On the other hand, in the x direction the person was stable all the time, as the ZMP location was within the boundaries of the support polygon throughout the whole measurement, as shown in the bottom plot of Fig. 5.

Fig. 6 shows the same support polygon as Fig. 5, but with the ZMP and the edges of the support polygon expressed as angles around the COM of the measured person, defined by (3) and (5), respectively. The angular results are similar to the results expressed with Cartesian coordinates on the ground, but the lines representing the edges of the angular support polygon are more curved than the corresponding lines in the Cartesian coordinate system, as the angular results are not obtained relative to a fixed coordinate system but relative to the COM of the moving person. The angular values of the ZMP are, on the other hand, subjected to smaller variations in time than the corresponding Cartesian values.

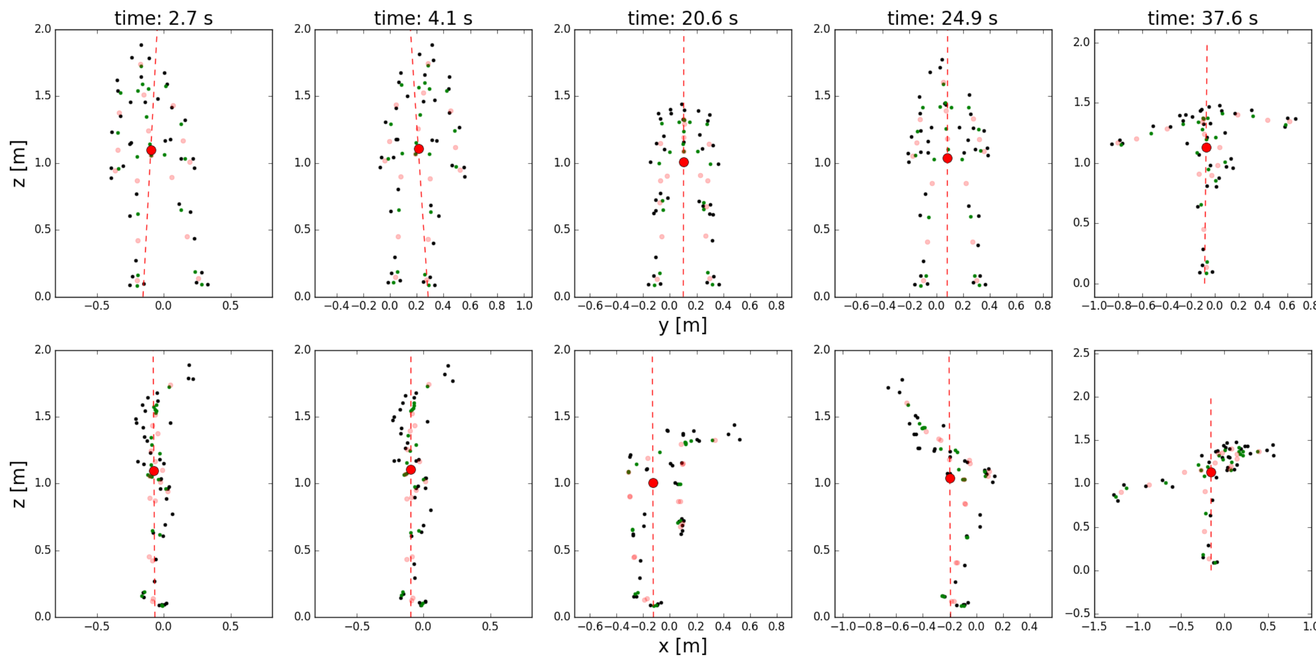

Fig. 4: Outtakes of the recorded and reconstructed data of the first filmed motion. The black dots represent the locations of the markers positioned on the filmed person, the green dots represent the reconstructed joint positions by the Motive software, the pale red dots represent the reconstructed locations of the COMs of each body link, the big red dot represents the location of the total COM of the filmed person and the red dotted line is the ZMP line. For the explanation of the body postures in each frame see the text.

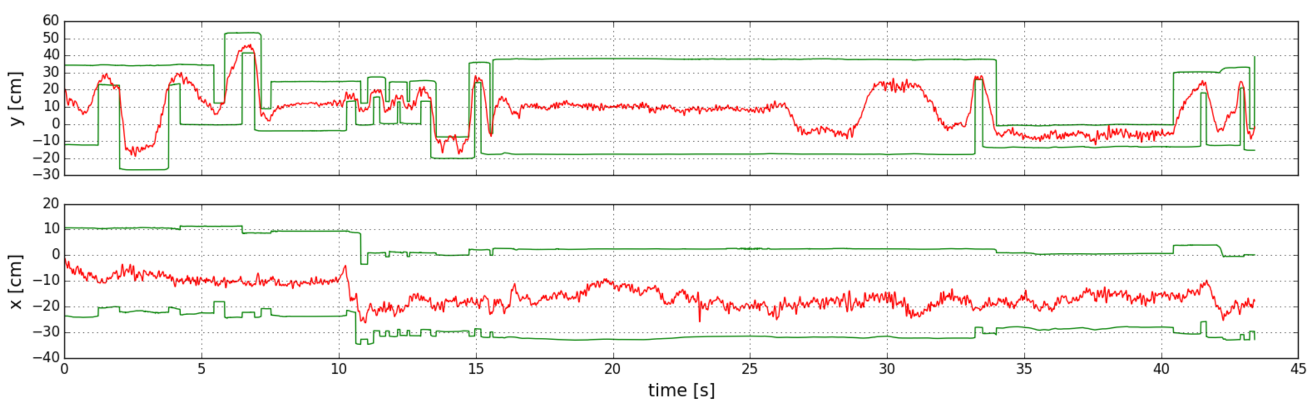

Fig. 5: The y (top) and x (bottom) coordinates of the ZMP (red lines) and the edges of the support polygon (green lines) for the first motion measurement expressed on the ground.

In the second motion filmed with the OptiTrack cameras the person is sitting down on a bench and standing up. Fig. 7 shows the frame sequence outtake of this motion.

In the first capture at time 0.2 s the person was stepping in front of the bench, in the second capture at 1.6 s the person was sitting down with his hands positioned on the bench, in the third capture at time 5.4 s the person was seated and in the forth and fifth captures at 8.2 s and 8.8 s, respectively, the person was standing up.

As throughout this motion the person was not supported only by his feet on the ground but also by his hands and bottom on the bench, the support polygon does not extend only on the ground but also on the bench. This is why the standard Cartesian definition of the ZMP and the support polygon on the ground can not be used but the angular redefinition of these quantities must be applied. Fig. 8 shows the angles of the ZMP and the edges of the support polygon obtained with (3) and (5), respectively. When the person was supported also by his hands and his bottom on the bench, the x coordinate of his hell from (5) was substituted with the x coordinate of his hand or bottom, while the height

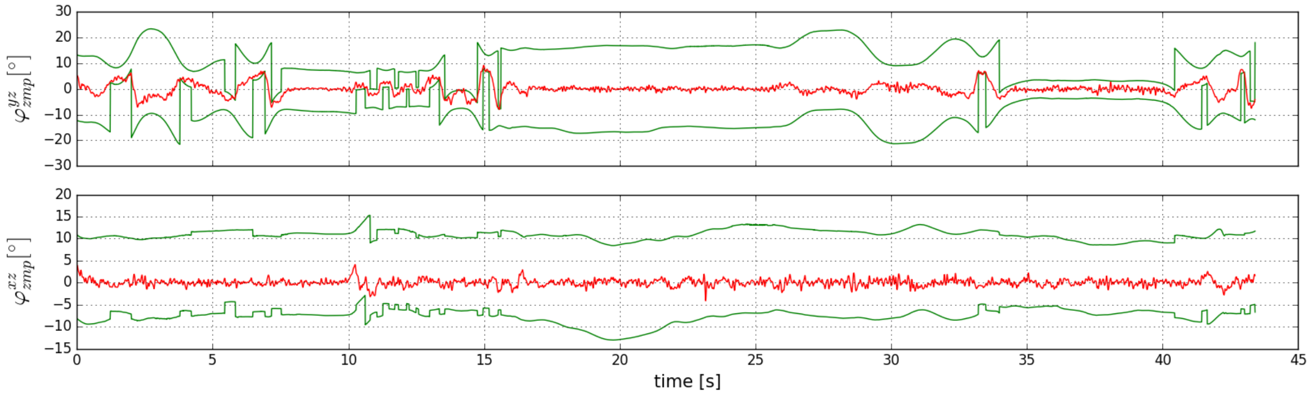

Fig. 6: The angles of the ZMP (red lines) and the edges of the support polygon (green lines) in the lateral (top) and the sagittal (bottom) planes of the measured person, for the first motion measurement

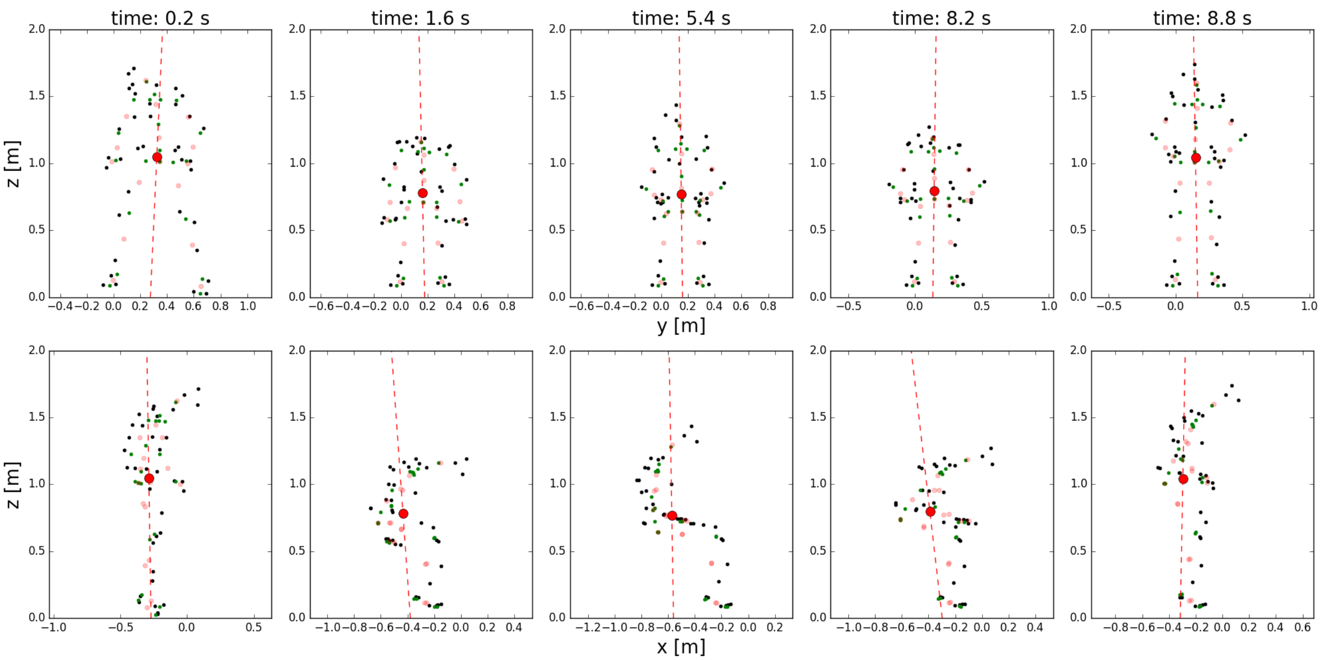

Fig. 7: Outtakes of the recorded and reconstructed data of the second filmed motion of sitting and standing. For the explanation of the meaning of different symbols see the text under Fig. 4. For the explanation of the body postures in each frame see the text.

of the COM was recalculated relatively to the bench height. In this measurement the ZMP was always within the support polygon which means that the person was stable all the time. In the beginning of the filming when the person started to sit down, he was supported only by his feet. At time 1.3 s he placed his fingers and at time 1.5 s also his palms on the bench, which can be seen as a widening of the support polygon angles in both the lateral and the sagittal planes, as the support polygon extended from his feet to the locations on the bench, where he was supported. At time 1.7 s the person sat on the bench and lifted up his arms

from the bench, which can be seen as a narrowing of the support polygon in the lateral plane and widening of the support polygon in the sagittal plane. The person was sited till the time 7.8 s, when he started to stand up which caused the narrowing of the support polygon in the sagittal plane, as the outermost location on his thighs where he was still in contact with the bench was approaching the edge of the bench above his feet. In the moment when he detached from the bench and was supported only by his feet at 8.1 s, the ZMP moved within the support polygon limited by the edges of his feet. Immediately after he was supported only by his feet the ZMP angle was in the sagittal plane close to the angle of his heel, but when he straightened up, the ZMP angle moved approximately in the middle between the angles

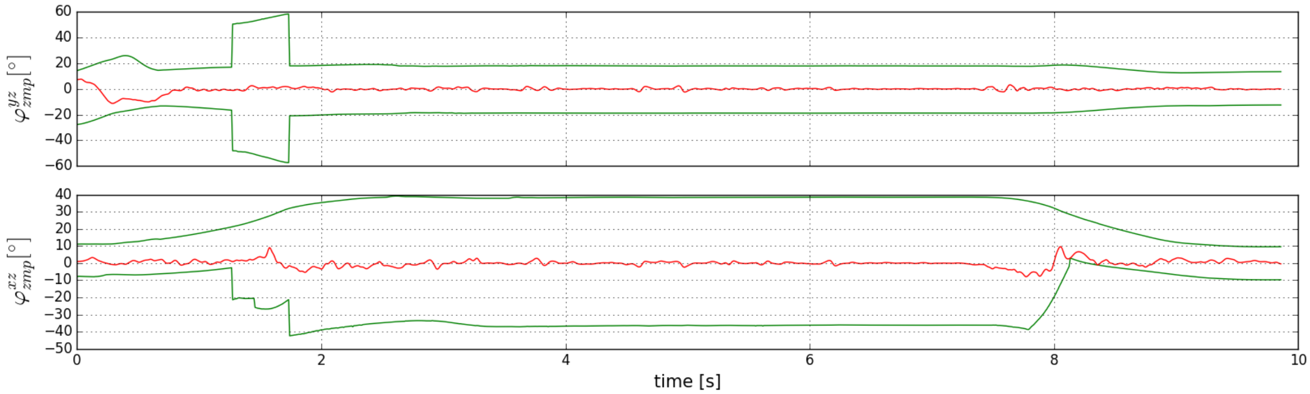

Fig. 8: The angles of the ZMP (red lines) and the edges of the support polygon (green lines) in the lateral (top) and the sagittal (bottom) planes of the measured person, for the second measurement of sitting and standing.

of his heels and his toes.

IV. CONCLUSIONS

The standard Cartesian definition of the ZMP on the ground is not suitable for the stability analysis in the situations when for example the ZMP lies at large distances from the investigated systems or if the system is supported at different heights. But in all these scenarios the angular definition of the ZMP can be used. If the system has support mechanisms that can act at different heights, such as for example a humanoid with arms, the situations when the ZMP would lie at far distances from the humanoid on the ground can be easily handled if the humanoid is supported at angles that embed the ZMP angle and if the friction forces and the forces in his joints allow for the satisfaction of the torque balance equation.

In this article it is shown how the stability of a person can be examined in the case when the person is supported only by his feet on the ground and in the case when the person is supported at different heights. In the first case both definitions of the ZMP and the support polygon were used, the one that defines these quantities in the Cartesian coordinates on the ground and the one that defines them as angles around the COM of the measured person. But in the second case, when the person was sitting on a bench and standing up, he was supported at different heights and the standard Cartesian definition of the ZMP and the support polygon on the ground could not be used and therefore our angular redefinition was applied. This way the stability of the person could be monitored in all the situations, no matter where the person was supported and whether the support mechanisms were only his feet or also his hands and his bottom. But the angular definitions of the ZMP and the support polygon are general and can therefore be applied to any system.

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