Performance Degradation of Coherent Direct Wideband Localization Due to Uncertainty in Receive Antenna Positions

Nenad Vukmirović, Miloš Janjić, and Miljko Erić

***Abstract*—In the context of passive coherent direct localization by a distributed receiving antenna array, we analyze how much the localization error increases due to non-ideal knowledge of receive antenna positions. We perform Monte-Carlo simulations with a wideband localization algorithm for a large distributed antenna array that surrounds the area were the transmitters are, and for an array of two pairs of antennas facing the area from a side. The former exhibits a very low increase in localization error, whereas the latter increases the error significantly, compared to the effect of the noise. We also derive approximate confidence intervals to confirm the validity of the drawn conclusions.**

***Index Terms*—Receive antenna position uncertainty; confidence intervals; coherent direct wideband position estimation; distributed antenna array; massive MIMO**

I. Introduction

IN this paper, we analyze a system that performs passive coherent direct wideband localization of a radio source (Tx) transmitting an arbitrarily wideband signal. The paper [1] explained the importance of wideband modeling, especially in newer generations of wireless systems. The (receive) antennas of the system are distributed in the area where the localization is performed. Therefore, we cannot assume planar wavefronts, but treat them as spherical, [2]. Each receive antenna is connected to an appropriate front-end, thus forming a single receive channel. Coherent localization requires that spatial coherence exists in the propagation medium and that the receive channels are time, frequency and phase synchronized, as described in [3], [4]. We assume that these conditions are satisfied. Even though the antennas are distributed, their front-ends could be held at the same place (collocated), such as in the example architecture in Fig. 1, which makes it easier to synchronize them. There are multiple sources of localization error in this scenario, such as the noise, interference, multipath propagation, synchronization errors and the uncertainty in the placement of the receive (Rx) antennas of the system, to mention a few.

The paper [3] showed that an error of about one thousandth of the carrier wavelength was achievable with coherent localization. A similar TDoA (Time Difference of Arrival) error (when converted to a length) was shown to be achievable in [5]. However, these results were obtained when the receive antenna positions were known exactly. The problem of accurate receive antenna placement is an important theoretical as well as practical problem, which the authors have encountered in a hardware implementation of a system for coherent localization, based on the methods in [3].

The impact of array element errors, either as (correlated) array shape distortions, independent errors of elements, or both, on the main beam direction, width, gain, as well as the sidelobe level was analyzed in [6]–[12]. The impact on direction of arrival estimation was analyzed in [13] and the impact on localization in [14]. Since the antennas in our paper are distributed, we model the element position errors as mutually independent and random. Also, since we are interested in localization of Txs inside the array aperture or close to it, the measure we use to quantify the impact of these errors is the localization accuracy.

In this paper, we are interested in the effects of the uncertainty in the Rx antenna positions on the accuracy of the mentioned type of localization. The authors of [15] proposed a method to estimate

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the position of a sensor in the array performing localization. The accuracy with which the antennas (the sensors) in the array are placed has to be greater in coherent radio localization, which this paper deals with, because, generally, the antennas have to be placed more accurately than the expected accuracy of the localization they are used for. Besides providing user location information for location-based services, the main purpose of coherent localization is to improve link performance in wireless systems, [16]. One such application of localization is in distributed beamforming. Distributed beamforming is robust with respect to the ambiguity problem, which exists in coherent localization (as explained on page 12 in [3]), so we do not focus on it in this paper. Many of the papers dealing with localization usually assume perfectly known Rx antenna positions, so it is important to analyze the effects of imperfect knowledge of Rx antenna positions. The results of this paper are important for selecting methods for measuring the geometrical relations (such as distances) between the antennas of the localization system, as well as for any applications that rely on accurate location estimation (with subwavelength accuracy) of radio transmitters.

II. Problem Formulation

Specifically, we want to quantify the increase in localization error due to the Rx antenna placement error compared to the noise-only scenario. To that end, we will use a signal model similar to that in [3], given by

$$u_m(t) = a_m \exp\left(-\mathrm{j}\omega_\mathrm{c}\left(t_0+\tau_m\right)\right) s\left(t-t_0-\tau_m\right) + \eta_m(t), \tag{1}$$

where $u_m(t)$ is the signal in Rx channel m, $m \in \{1, 2, \ldots, M\}$, M is the number of the Rx antennas, a_m is a real-valued attenuation coefficient, $\omega_\mathrm{c} = 2\pi f_\mathrm{c}$ is the carrier frequency, t_0 is the unknown shift between the Tx and Rx time axis, $\tau_m = d_m/c$ is the propagation delay from the Tx, at an unknown position $\vec{r}$, to Rx antenna m, at $\vec{r}_m$, c is the speed of propagation, $d_m = \|\vec{r} - \vec{r}_m\|$, and $\eta_m(t)$ is the complex additive white Gaussian noise, AWGN. Figure 1 shows the system model and geometrical relations between the antennas. Note that, in coherent localization, the phase term contains the carrier phase only, and is modeled (by

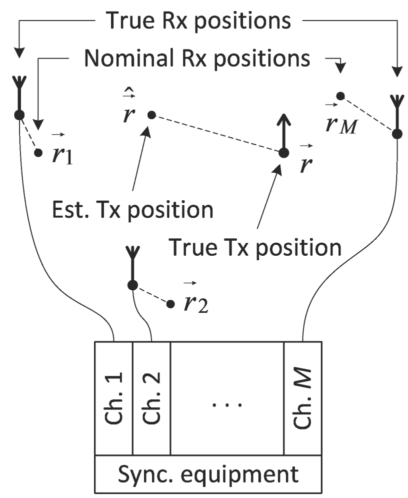

Fig. 1. The system model.

the exponential term) separately from the amplitude a_m. The term $s\left(t-t_0-\tau_m\right)$ models the envelope time delay of the transmitted signal/sequence $s(t)$ (wideband modeling). The signal-to-noise ratio, SNR, in channel m is $\mathrm{SNR}_m = \mathrm{SNR}_0 d_0^2/d_m^2$, where SNR_0 is used as a reference SNR at a distance of $d_0 = 1$ m. For convenience, let the unit of time be one sampling interval. This does *not* mean that the time variables/parameters are integers.

In each channel m, the samples available to a localization algorithm are $u_m(t)$, for $t \in \{0, 1, \ldots, N-1\}$. The algorithm computes an estimate of the Tx position, $\widehat{\vec{r}}$, with an error $\Delta r = \|\widehat{\vec{r}} - \vec{r}\|$. The root-mean-squared localization error is

$$\mathrm{RMSE} = \sqrt{\mathrm{MSE}} = \sqrt{\mathrm{E}\,\Delta r^2}. \tag{2}$$

Since the Rx antenna positions, $\vec{r}_m$, are random variables, we can define the receive antenna RMSE, the RxRMSE, similarly. Since different methods can be used to position the Rx antennas, we use a generic model for the position errors. Let us assume that the error of each $\vec{r}_m$ along each of the axes (x and y; if antennas are distributed in 3D, then also z) is an independent 0-mean Gaussian random variable with the same variance σ_ax^2. Therefore RxRMSE is either $\sigma_\mathrm{ax}\sqrt{2}$ for 2D or $\sigma_\mathrm{ax}\sqrt{3}$ for 3D localization. The goal is to analyze how much the RMSE increases with RxRMSE.

III. Localization with Antenna Position Uncertainty: Error Analysis

We estimated the RMSE (which contained the effects of both the noise and the Rx antenna position uncertainty) through Monte-Carlo simulations for segments of random Gaussian transmitted sequences that were $N = 256$ samples long. The carrier frequency was 60 GHz and the bandwidth was 100 MHz. We used the SCM-MUSIC from [3] as a representative of coherent localization methods for unknown transmitted sequences. For a given set of Rx antenna positions, i.e. the array geometry, the true positions were randomly generated for each simulation run according to the given RxRMSE.

A. Approximate Confidence Intervals

We will characterize the quality of an estimate of the localization RMSE by an approximate confidence interval. To that end, we first note the distribution of the estimator $\widehat{\text{MSE}}$. We have that $\widehat{\text{MSE}}$ is the mean of the values Δr_k^2, $k \in \{1, 2, \ldots, K\}$, where K is the number of Monte-Carlo runs and Δr_k is the Euclidean distance between the estimated and the true location of the transmitter (i.e. the location error). The values Δr_k^2 are i.i.d. random variables, so, according to the Central limit theorem, the distribution of $\left(\widehat{\text{MSE}} - \mu\right)/\sigma$ is $\mathcal{N}(0,1)$, where μ and σ^2 are the mean and variance of $\widehat{\text{MSE}}$, respectively. Further, we have $\mu = \text{E}\,\Delta r_k^2 = \text{MSE}$ (the true value) and $\sigma^2 = \text{Var}\,\Delta r_k^2 / K$.

For a given level of confidence, p, define ε as $\mathcal{P}(|\xi| \le \varepsilon) = p$, where $\xi \sim \mathcal{N}(0,1)$. Therefore,

$$\varepsilon = \sqrt{2}\,\text{erfc}^{-1}(1-p)\,. \tag{3}$$

Next, recall that

$$\Delta r_k^2 = \Delta x_k^2 + \Delta y_k^2 + \Delta z_k^2, \tag{4}$$

where Δx_k, Δy_k, and Δz_k are the individual errors along the coordinate system axes (for 2D localization $\Delta z_k = 0$). We assume that they are 0-mean Gaussian random variables with an unknown level of correlation. Thus, Δr_k^2 is expected to have a chi-square distribution with n degrees of freedom, $\chi^2(n)$, where we expect $n = 2$ in the case when the Tx is inside the aperture of the array for 2D localization, $n = 3$ inside the aperture for 3D localization, and $n = 1$ when the individual errors are highly correlated or when one of them is dominant over the other two (when the Tx is outside the array aperture). For a random variable $W_n \sim \chi^2(n)$, we rely on

$$\text{Var}\,W_n = \frac{2}{n}\left(\text{E}\,W_n\right)^2, \tag{5}$$

as a property of the chi-square distribution. Combining the previous properties, we obtain

$$\mathcal{P}\left(\left|\widehat{\text{MSE}} - \text{MSE}\right| \le d\right) = p, \tag{6}$$

where

$$d = \varepsilon\sqrt{\frac{\text{Var}\,\Delta r_k^2}{K}} = \text{MSE}\cdot\varepsilon\sqrt{\frac{2}{nK}}. \tag{7}$$

To approximate this, we use $n = 1$ as the worst case (the one that produces the widest confidence interval) and, since MSE is unknown, we use $\widehat{\text{MSE}}$ instead:

$$\widehat{d} = \widehat{\text{MSE}}\cdot\varepsilon\sqrt{\frac{2}{K}}. \tag{8}$$

Instead of using an absolute confidence interval $\left[\widehat{\text{MSE}} - \widehat{d}, \widehat{\text{MSE}} + \widehat{d}\right]$, we can use a relative one, $\left[\widehat{\text{MSE}}/\delta^2, \widehat{\text{MSE}}\cdot\delta^2\right]$, by defining δ as $\widehat{\text{MSE}}/\delta^2 = \widehat{\text{MSE}} - \widehat{d}$. This produces an approximate confidence interval for the $\widehat{\text{RMSE}} = \sqrt{\widehat{\text{MSE}}}$,

$$\left[\widehat{\text{RMSE}}/\delta, \widehat{\text{RMSE}}\cdot\delta\right], \tag{9}$$

where

$$\delta = \sqrt{\frac{1}{1 - \varepsilon\sqrt{\frac{2}{K}}}} = \left(1 - \frac{2}{\sqrt{K}}\,\text{erfc}^{-1}(1-p)\right)^{-1/2}. \tag{10}$$

Note that one convenient property of δ is that it does not depend on either the geometry, or the value of the MSE. It only depends on the number of simulation runs, K, and the chosen level of confidence, p. This formula is also useful for determining the number of simulation runs needed for a confidence interval of a given width.

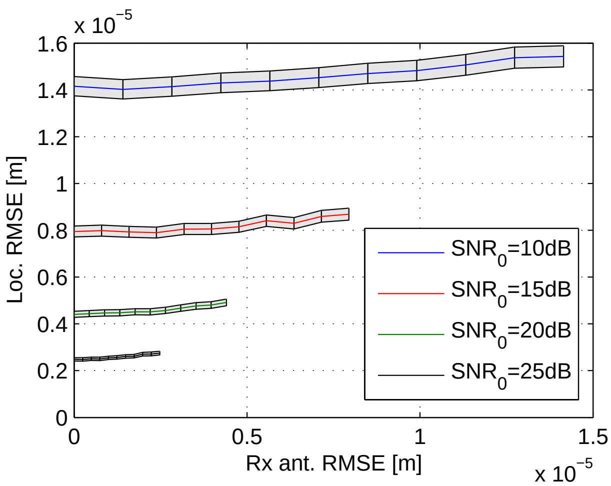

Fig. 2. The localization RMSE vs RxRMSE for different values of SNR_0 for the array geometry G_{18}.

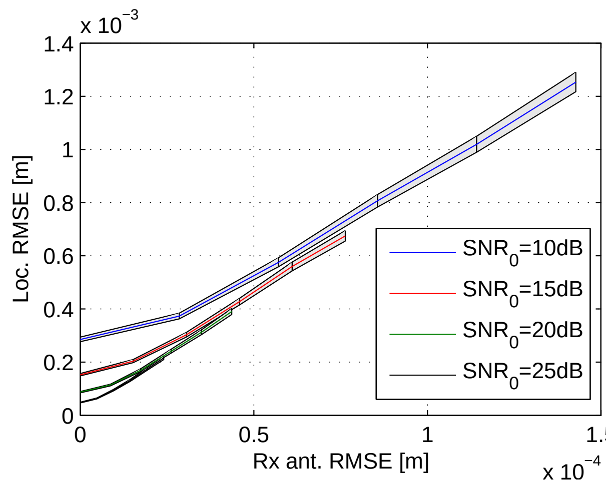

Fig. 3. The localization RMSE vs RxRMSE for different values of SNR_0 for the array geometry G_4.

B. Simulation Results

The results for different SNRs were generated for $K = 4096$ runs. The width of the confidence intervals (9), for $p = 0.99$, was then determined by $\delta = 1.03$, (10).

Figure 2 shows the results for a geometry G_{18}, used in [3], and based on [17], for localization in the horizontal plane 1.2 m below the array. The Tx was roughly below the center of G_{18}. The SNRs in the channels were grouped around the value 6 dB below SNR_0. The figure shows four curves for SNR_0 values of 10, 15, 20, and 25 dB. Note that they were evaluated only for RxRMSE below the RMSE of localization when there is no Rx antenna uncertainty, because it only makes sense that the accuracy of Rx antenna placement is greater than the accuracy of the localization method for the Tx. The curves show a very low increase in localization RMSE with an increase of RxRMSE. This can be explained by the fact that the number of Rx antennas is relatively large, they surround the area where the Tx is expected to be, and the placement errors are independent, so that all antennas would not move the main lobe of the localization algorithm in the same direction. Instead, those errors tend to partially cancel each other out, so the dispersion of the maximum of the main lobe is increased only slightly.

On the other hand, if some of the Rx antennas are close to each other (the antennas are grouped into subarrays), the number of antennas is small, or they do not surround the area where the Tx is, a much larger increase in RMSE is expected. Figure 3 confirms this and shows the results for a geometry G_4 with two subarrays of two antennas each, with their broadsides facing the area in front of the array. The (x, y) coordinates in [m] of the Rx antenna positions were $(0.0884, -0.0884)$, $(-0.0884, 0.0884)$, $(2.5316, -0.0884)$, and $(2.7084, 0.0884)$. The Tx was placed in front of the array at $(1.3, 1.5)$ in [m] and the distance to the Rx antennas was around 2 m. This means that the actual SNRs were 6 dB below SNR_0. To make the comparison fair, the curves were then evaluated for the same values of SNR_0 as for the G_{18} case. The increase in RMSE for the maximum mentioned RxRMSE was very large (around 9 times) so the curves are only shown for values below one half of that.

It would also be worthwhile to explore how the localization RMSE scales with the number of Rx antennas, m. However, to make the comparison for different values of m fair, for each m the array geometry should be optimized in some way. If the geometries were deterministic, it is unlikely that the geometry for m can be generated form the geometry for $m - 1$ by adding a single antenna without changing the positions of the others. If the geometries were random, the RMSE values should be averaged over different realizations of these geometries for all considered m. A uniform circular array could be considered for a fair comparison, but there are practical limitations that need to be considered as well, e.g., the antennas would probably be placed on walls or possibly on the ceiling of a room (in which the localization occurs), so this constrains the positions of the antennas to a rectangle/cuboid. However, since strict optimization of the Rx array geometry is outside the scope of the paper, this remains as an interesting topic for future research.

IV. Conclusion

We analyzed the impact of receive antenna position uncertainty on the accuracy of coherent direct wideband localization by a distributed receiving antenna array. Independent Gaussian errors in receive antenna positions were assumed. According to the simulation results, the impact of this uncertainty is small compared to the effect of the noise for G_{18}, which has a large number of antennas that encompass the area where the transmitters are. On the other hand, for arrays which have a small number of antennas, or have closely packed subarrays of antennas, especially when the transmitter is outside the aperture, such as G_4, the degradation of localization accuracy is rather large. The confidence intervals show that these effects are due to the Rx antenna positions uncertainty, and not merely due to the randomness in the simulations.

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