Resolvability of Transmitters in Coherent Direct Localization

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***Abstract*—Coherent direct localization promises high accuracies, that are especially useful for improving wireless link performance (the location-aided communication concept). The focus of the paper is to analyze the spatial resolution performance of three different localization algorithms in this category in the context of spectrum sensing – i.e., their ability to successfully resolve multiple transmitters at different positions working in the same band and time interval. Namely, when two transmitters are close to each other, they interfere with the localization process, which can perceive them as a single source (and, therefore, fail to resolve them). We quantify the impact of this interference on the probability of resolution and the localization error for both cooperative and non-cooperative transmitters. The results of simulations show that, even when the distance between the transmitters is lower than the carrier wavelength, given that the inherent ambiguity problem allows, they can be resolved, with a localization error of a small fraction of the wavelength. The resolution rate is extremely high for the algorithm with a priori known waveform (for cooperative transmitters).**

***Index Terms*—Coherent direct position estimation; distributed antenna array; resolvability of multiple transmitters; spectrum sensing**

I. Introduction

THE focus of the paper is an analysis of spatial resolution performance of coherent localization methods in the context of spectrum sensing. Spatial resolution refers to the ability of an algorithm to correctly perceive two signal sources that are close to each other as two different sources and to estimate their positions, based on the received signals.

Resolution of two known waveforms in noise is analyzed in [1], such as two complex sinusoids of similar frequencies. Resolution performance of direction-of-arrival estimation for sources by a collocated sensor array is assessed in [2]. The paper [3] generalizes the resolution analysis to multiple parameters per signal (such as the spatial coordinates of its source) and multiple signals. The impact of blurring on the resolution in image-forming applications is provided in [4].

The authors of [5] discuss different criteria for successful resolution of acoustic sources for different methods. They propose the valley-to-peak ratio (VPR) as a measure of the quality of resolution. Namely, if a localization method has a criterion function whose maxima represent the estimated positions of the sources and there are two sources of equal intensity close to each other, then their maxima and the minimum between them define the VPR. Our paper is based on simulations in which the maxima corresponding to two radio transmitters are searched for starting at their true positions. If the two search instances (one for each transmitter) end at the same point, it is considered that the transmitters have not been resolved in that attempt. We quantify the performance of resolution by the probability of success and the impact on (deterioration of) the position estimation accuracy. If the VPR is low, the noise and interference have a greater chance of making the resolution process fail (we implicitly rely on the VPR for quantification). Additionally, we generalize the analysis to transmitters of different power levels.

II. Problem Formulation

Let us consider a distributed array of M receive (Rx) antennas at known positions $\vec{r}_m$, $m \in \{1, 2, \dots, M\}$. The Rx antennas are placed in an area where localization of transmitters (Tx) is performed. We analyze the performance of *coherent* localization. This type of localization requires a propagation medium in which the spatial coherence

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condition holds for (at least) the line-of-sight (LoS) components of the signals which the Txs transmit and the Rx array receives. This condition, [6], [7], allows us to use the additional information embedded in the carrier phases of the signals, to increase the accuracy, unlike non-coherent methods.

The raw received signals are processed by the system performing the localization (therefore the localization is *direct*). Obviously, the receiving channels (the Rx antennas, the Rx front-ends, and the signal cables between them) need to be time, frequency, and phase synchronized (t-, f-, and φ-sync). This can be achieved by means of hardware calibration or by processing beacon signals from a dedicated anchor (say, a base station). The baseband complex form of the signal each Rx channel m receives is

$$\begin{aligned} u_m(t) =& \eta_m(t) + \sum_{q=1}^{Q} s_m^{(q)}(t) \\ s_m^{(q)}(t) =& A_m^{(q)} e^{-\mathrm{j}\omega_\mathrm{c}\left(t_0^{(q)}+\tau_m^{(q)}\right)} s^{(q)}\left(t - t_0^{(q)} - \tau_m^{(q)}\right), \end{aligned} \tag{1}$$

where $\eta_m(t) \sim \mathcal{CN}\left(0, \sigma^2\right)$ is an independent white Gaussian noise; Q is the number of Txs; $s^{(q)}(t)$ is the waveform of Tx_q; $t_0^{(q)}$ models the lack of t-sync between Tx_q and the Rx system; $\tau_m^{(q)}$ is the propagation time from Tx_q to Rx_m and $A_m^{(q)}$ is the amplitude factor; the carrier phase is in the exponent and $\omega_\mathrm{c} = 2\pi f_\mathrm{c}$ is the carrier frequency. In this model, the frequencies are normalized by the sampling frequency, $\widetilde{f}_\mathrm{s}$, and time values by $\widetilde{f}_\mathrm{s}^{-1}$, e.g. $f_\mathrm{c} = \widetilde{f}_\mathrm{c}/\widetilde{f}_\mathrm{s}$, $t = \widetilde{t}\widetilde{f}_\mathrm{s}$, $\tau_m^{(q)} = \widetilde{\tau}_m^{(q)}\widetilde{f}_\mathrm{s}$, and so on, where the symbol $\sim$ denotes values in physical units (Hz and s). To keep the analysis tractable, we restrict it to the LoS-only scenario. Then the coherence implies that $A_m^{(q)}$ is real valued.

Specifically, we are interested in analyzing the ability of localization algorithms to distinguish between different Txs which transmit in the same band and time interval (the ability to resolve them successfully) even if they are close to each other. A localization algorithm produces an estimate, $\widehat{\vec{r}^{(q)}}$, of the true (and unknown) location $\vec{r}^{(q)}$ of Tx_q. If, say, $\widehat{\vec{r}^{(1)}} = \widehat{\vec{r}^{(2)}}$ (to within the numerical precision), than the algorithm has failed to resolve the different transmitters Tx_1 and Tx_2. We wish to find the minimum distance between them at which they are still resolvable.

III. The Method

Unlike a system with a single classical (collocated) antenna array, which can estimate the direction of arrival of an incoming radio signal, a system with antennas distributed around the Tx area can estimate their positions, even if they are not t-synchronized with it. We perform Monte-Carlo simulations of such a scenario with $Q = 2$ Txs. We cover the inside of the Rx array aperture by a discrete set of nominal Tx points (the Tx grid). This allows us to average the results over the space. In each simulation run, the positions of Tx_1 and Tx_2 are generated with a specified distance between them and a random orientation at one of the Tx grid points.

A direct localization algorithm in this paper has a criterion function g defined over the area of interest. The only difference between g and a cost function is that a cost function is searched for its minima, whereas g is searched for its maxima. The search is initialized for each of the two Txs at its true location, $\vec{r}^{(q)}$, and it follows the gradient of g to find the maximum, which is the estimate of that Tx's location, $\widehat{\vec{r}^{(q)}}$. Multiple runs are performed at each Tx grid point to achieve a desired statistical sample size. Successful Tx resolutions are counted and the squared Euclidean distance between the estimated and true location of a Tx, $\|\widehat{\vec{r}^{(q)}} - \vec{r}^{(q)}\|^2$, is averaged. Thus, we obtain an estimate of the probability of resolution and the root-mean-square-error (RMSE) of localization for that algorithm.

We use the ML-KS (maximum likelihood – known sequence), ML-US (ML – unknown sequence), and SCM-MUSIC (steered-covariance-matrix multiple-signal classification) algorithms from [6] as representatives of coherent algorithms. ML-KS has stricter requirements for the Tx than the other two. It requires that the modulator in the Tx is coupled with its D/A converter so that the carrier phase is 0 at $t = 0$ (on the local time axis) for each processed signal segment. ML-KS also needs to know the Tx's waveform. This is suitable for localization of cooperative Txs, such as user terminals (UT) in a wireless network, where the base stations allocate training waveforms for the UTs and also perform localization.

ML-US and SCM-MUSIC impose no such restrictions and are suitable even for non-cooperative radio sources. For them, deviations in the modulator phase coupling and carrier frequency can be considered as a part of the Tx waveform itself (since it is unknown to the Rx system, anyway).

A. Grating Lobes

Coherent algorithms suffer from the (integer wavelength) ambiguity problem. It can be intuitively explained like this. If the system performs distributed beamforming in the downlink, than there might appear spots in the area other than the UT antenna location where the electric field vector also has an increased intensity. Localization based on uplink signals would then have high lobes in its criterion function at those spots (the sidelobes), not only at the true UT location (the main lobe).

When analyzing Tx resolvability, we have to consider not only the distance between the Tx_1's main lobe and that of the Tx_2, but also to the closest high sidelobe (the closest grating lobe) of Tx_2. This increases the chance the Txs will interfere with each other. However, if they are moving, it is expected that the overlapping of the lobes will happen only for very short periods of time, so that they would be resolvable most of the time. This definitely seems like an important topic for future research.

IV. Simulation Results

Let us define SNR_0 as the signal-to-noise ratio (SNR) of a Tx's signal in a channel whose Rx antenna would be 1 m away from the Tx. We performed Monte-Carlo simulations with $Q = 2$ Txs, where we kept the SNR_0 of Tx_2 at 30 dB. The (power) level of Tx_1 was 0 dB, -5 dB, and -15 dB relative to Tx_2. The Rx array had 5 antennas at (x, y) coordinates $(-2.195, -1.243)$, $(0.177, -2.641)$, $(2.961, -1.056)$, $(2.534, 2.206)$, and $(-2.18, 2.237)$ in [m]. Each (Tx and Rx) antenna was assumed to have an omnidirectional radiation pattern in the plane of the array. The area inside the array's aperture was covered by a Tx grid with 28×28 points. For each point we performed $K = 3$ simulation runs. In each run, Tx_1 was placed at the corresponding Tx grid point and Tx_2 was placed randomly (with uniform distribution) on a circle centered at Tx_1 with the radius equal to the given distance between Tx_1 and Tx_2, denoted by $d_{12} = \|\vec{r}^{(1)} - \vec{r}^{(2)}\|$. The waveform of each Tx in each run was generated based on a new independent realization of a random complex Gaussian sequence of $N = 256$ samples, with $\widetilde{f}_c = 1$ GHz and $\widetilde{f}_s = 10$ MSample/s.

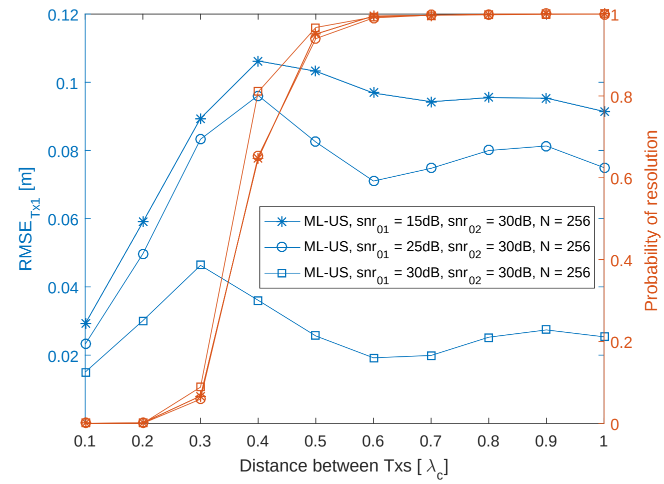

Fig. 1. Probability of resolution and RMSE vs. d_{12} for the ML-US algorithm.

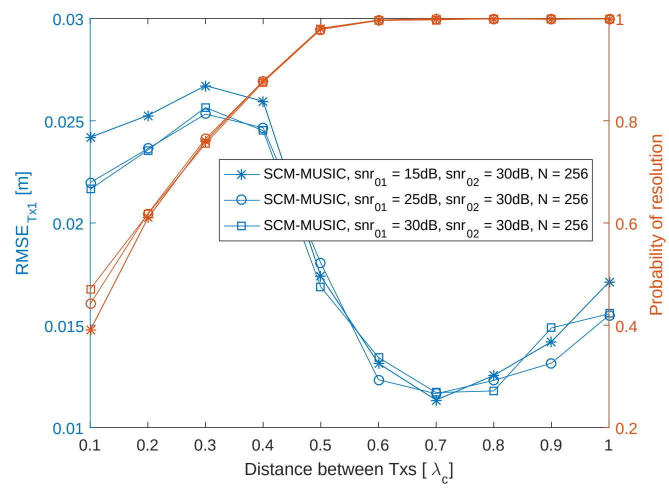

Fig. 2. Probability of resolution and RMSE vs. d_{12} for the SCM-MUSIC algorithm.

The results of simulations for the ML-US algorithm vs. the distance between the Txs (given in carrier wavelengths, λ_c) are shown in Fig. 1. The algorithm resolves the Txs in most cases when $d_{12} \geq 0.5\lambda_c$ and fails in most cases when $d_{12} \leq 0.3\lambda_c$. The RMSE for Tx_1 (the Tx with lower or equal power) is approximately in the range 2 cm – 10 cm when the Txs are successfully resolved. Then the RMSE deteriorates with decreasing d_{12}.

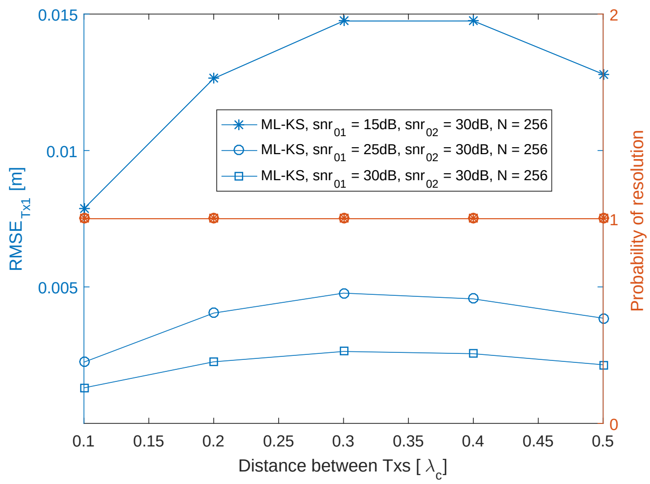

Fig. 3. Probability of resolution and RMSE vs. d_{12} for the ML-KS algorithm.

Note that the RMSE appears to improve for very low d_{12}, but that is just a consequence of Tx_1 location estimate being close to Tx_2, which itself is closer and closer to Tx_1 (there is no actual improvement).

The results for SCM-MUSIC are depicted in Fig. 2. This algorithm has better resolvability in the low-d_{12} region (below $0.5\lambda_c$) than ML-US. Furthermore, all three RMSE curves are similar to the ML-US curve for the best-case scenario of the given three regarding the difference in levels between Tx_1 and Tx_2 (when they transmit at an equal power), so we conclude that SCM-MUSIC is robust with respect to this difference. Better performance of this algorithm is expected, since it is based on a high-resolution algorithm (the generic MUSIC), although, this comes at the cost of higher numerical complexity.

The results for ML-KS are shown in Fig. 3. We can see that it has an extremely high resolvability across the d_{12} range (nearly 1), owing to the fact that the sequences (and waveforms, as well) of the Txs are nearly orthogonal with high probability (they are independent random vectors). We also see that the RMSE is quite low – in most cases it is below 1 cm and it is not affected much by d_{12}. This comes at a price of increased numerical complexity and the reduced scope of applications (as explained in the previous text) – the algorithm has to know the Tx's sequence, so it is mostly for cooperative applications.

One direction for future research is analyzing the impact of different levels of orthogonality between the Txs' waveforms on localization performance. Another is optimization of the Rx antenna array's geometry to suppress the ambiguity problem, effectively reducing the chance the Txs would interfere with each other in the localization process. It would also be interesting to quantify the effect of the ambiguity on localization and tracking, when the Tx is moving.

V. Conclusion

In this paper we presented an analysis of coherent localization performance of multiple transmitters in the same band and time interval, for three different algorithms. The SCM-MUSIC algorithm performs better than ML-US in unfavorable conditions and has the same scope of applications, but at a higher numerical cost. The ML-KS has the best performance, but at a higher numerical cost and it is usually restricted to localization of cooperative transmitters. All in all, each of the analyzed algorithms have a localization error that is a small fraction of the carrier wavelength (as long as the ambiguity problem does not cause them to fail), despite the fact that the transmitters interfere with each other.

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