The Improved GM PHD algorithm for Multi-Target Radar Tracking

Zvonko Radosavljević, Dejan Ivković and Branko Kovačević

***Abstract*—Gaussian mixture probability hypothesis density (GM PHD) is a modern nonlinear algorithm for tracking multiple targets in a clutter environment. It is accompanied by known problems that are primarily related to the impossibility of associating the measurement of existing targets and determining the quality of the tracks. For this purpose the automatics track initialization by known 'two point initialization' was introduced. Difference of successive measurements from two radar antenna revolutions is compared with the threshold, which depends on the velocity of the target. The paper proposes to improve the algorithm by introducing the probability of the existence of a target and to reject false tracks. The results of intensive simulations of tracking multiple radar targets have shown the justification for the application of the proposed algorithms.**

***Index Terms*—Target tracking, probability hypothesis density, track while scan (TWS) radars.**

I. Introduction

Multi-target tracking in clutter, assuming linear target trajectory propagation and linear target measurement equation, naturally leads to a Gaussian mixture (GM) target tracking solution. As the origin of measurements is uncertain, both true tracks (which follow targets) and false tracks (which do not) exist [1].

The random finite sets (RFS) are representations of multi-target states and multi-target measurements. The modeling of multi-target dynamics using random sets leads to algorithms which incorporate track initiation and termination, a procedure that has mostly been performed separately in tracking algorithms. The first systematic treatment of multi-sensor multi-target filtering, as part of a unified framework for data fusion using random set theory was finite set statistics (FISST). The alternative to optimal multi-target filtering is the Probability Hypothesis Density (PHD) filter [2], [3], [4]. It is a recursion propagating the 1st moment, called the intensity function or PHD, associated with the multi-target posterior.

A Gaussian mixture, consisting of a weighted sum of Gaussian PDF, each with different means and covariance's, is the natural form of the PDF of target state. Using such a structure, a mixture component is created for every possible association, using every possible pairing of targets and measurements with the mean and covariance calculated assuming that the particular hypothesis is true, and the weight calculated to represent the probability that the particular hypothesis is true. The serious problem in multi-target tracking is the unknown association of measurements with appropriate targets [5], [6]. Moreover, the data association problem makes up the growth of the computational load in multi target tracking algorithms. Recently, multi-target tracking formulations involve explicit associations between measurements and targets

The Gaussian Mixture Probability Hypothesis Density Filter (GM-PHD Filter) provided a closed form solution to the PHD filter recursion for multiple target tracking [7]. The posterior intensity function is estimated by a sum of weighted Gaussian components whose means, weights and covariances can be propagated analytically in time. In particular, the means and covariances are propagated by the Kalman filter.

The original Gaussian Mixture PHD filter algorithm provided a means of estimating the number of targets and their states at each point in time. The method for determining the targets simply used the weights of the Gaussian components and did not take into account tempo al continuity. We show that if a target is not detected for each iteration, the Gaussian components can still track the targets in the presence of some missed detections. The trajectory of the target in the past, before it has been detected, can also be determined by keeping the trajectories of each of the Gaussian components. The original formulation of the GM PHD filter allowed targets to be spawned from existing targets [8].

The paper proposes and tests the improved GMPHD algorithm with automatic track initialization (ATI) via 'two point methodology'. Each incoming measurement (from the previous scan) is paired with each incoming measurement from the current scan in order to examine the possibility of initializing a new trace based on the knowledge of the maximum speed of movement of the targets [9].

Rest of papers is organized as follows. Second chapter is dedicated basic problem statements, which precedes Third chapter, which brief decrypted GM PHD algorithm. Results of experiments are given by Fourth chapter. Final conclusions are given by the Fifth chapter.

II. Problem Statements

Consider the target tracking scenario with two dimensions. Also, consider the tracking algorithm with two parameters:

Zvonko Radosavljević and Dejan Ivković is with the Millitary Technical Institute, Belgrade, Ratka Resanovica bb, 11030 Belgrade, Serbia (e-mail: zvonko.radosavljevic@gmail.com and divkovic555@)gmail.com).

Branko Kovačević is with the School of Electrical Engineering, University of Belgrade, 73 Bulevar kralja Aleksandra, 11020 Belgrade, Serbia (e-mail: kovacevic_b@etf.rs).

 ISBN 978-86-7466-930-3

probability of detection (P_D) and clutter density. The clutter density is depending on target dynamics and characteristic of sensor. Generally, clutter is defined by number of selection measurement from size of selection gate. At begin, consider the target state $x_k \in R^{n_z}$ at time interval *k*. The dynamic target trajectory state models at the time *k* are given by the:

$$x_k = Fx_{k-1} + v_k \tag{1}$$

where F is the propagation matrix, v_k is a zero mean and white Gaussian sequence with covariance *R*. At each scan the sensor returns a random number of random target and clutter measurements. At time *k*, one sensor delivers a set of measurements $z_k = \{z_{k,j}\}_{j=1}^{M_k}$ track out of which a set of measurements are selected for track update. Converted target measurement *y* is given by [10] :

$$y_k = Hx_k + w_k \tag{2}$$

where *H* is measurements matrix and the measurements noise w_k is zero mean and white Gaussian sequence. A measurements of target is present in each scan with a probability of detection P_D . Clutter measurements follow the Poisson distribution characterized at location by clutter measurements density $\rho_k(y)$ [11].

At time k a set of $m(k)$ measurements $Y(k) = \{y_i(k)\}_{i=1}^{m(k)}$ are detected, where each measurement either originate from one of *n* known linear measurement models or is a false detection. The sequences $v(k)$ are mutually independent and uncorrelated with the process noise.

A. Finite Sets statistics

Finite set statistics is the concept of belief-mass functions, which are non adaptive generalizations of probability mass functions and are equivalent to probability mass function on certain abstract topological spaces. The multi-target state and multi-target measurement at time k are represented as finite subsets X_k and Z_k, respectively.

The models of motion of the multi-target system using a randomly varying RFS is given by the [12]:

$$\Gamma_k = \Phi_k[X_{k-1}, V_{k-1}] \cup B_k[X_{k-1}] \tag{3}$$

where $\Phi_k[X_{k-1}]$ represents the dynamics of the existing targets, X_{k-1} is the random set of the state vectors of the random number of targets, and V_{k-1} denotes the system process noise, while $B_k[X_{k-1}]$ represents the process of target birth.

III. The Gaussian Mixture PHD Filter Algorithm

In this section, we describe the linear-Gaussian multiple target model and the recently developed Gaussian Mixture PHD filter. The multiple target models for the PHD recursion is described here. Each target follows a linear Gaussian dynamical model [13],

$$f_{k|k-1}(x|\xi) = N[x; F_{k-1}(\xi), Q_{k-1}] \tag{4}$$

$$g_k(y|x) = N[y; H_k(x), R_k] \tag{5}$$

where *N* (.;*m*,*P*) denotes a Gaussian density with mean *m* and covariance *P*, F_{k-1} is the state transition matrix, Q_{k-1} is the process noise covariance, H_k is the observation matrix and R_k is the observation noise covariance. The survival $p_{Sk}(x) = p_{S,k}$ and detection $p_{Dk}(x) = p_{D,k}$ probabilities are state independent. The intensities of the spontaneous birth and spawned targets are Gaussian mixtures,

$$\gamma_k(x) = \sum_{i=1}^{J_\gamma(k)} w_{\gamma_k}^i N[x; m_{\gamma_k}^i, P_{\gamma_k}^i] \tag{6}$$

$$\beta_{k|k-1}(x|\varsigma) = \sum_{j=1}^{J_\beta(k)} w_{\beta_k}^j N[x; F_{\beta_{k-1}}^j(\varsigma) + d_{\beta_{k-1}}^j Q_{\beta_{k-1}}^j] \tag{7}$$

where $J_{\gamma_k}, w_{\gamma_k}^i, m_{\gamma_k}^i, P_{\gamma_k}^i,\ i = 1,..., J_\gamma(k)$ are given model parameters that determine the shape of the birth intensity, similarly, $J_{\beta_k}, w_{\beta_k}^j, F_{\beta_{k-1}}^j, d_{\beta_{k-1}}^j$ and $Q_{\beta_{k-1}}^j$ $j = 1,..., J_\beta(k)$ determine the shape of the spawning intensity of a target with previous state.

A. Algorithm steps

1 Prediction step

Under the assumptions that each target follows a linear Gaussian dynamical model, the survival and detection probabilities are constant, the intensities of the birth and spawned targets are Gaussian mixtures, and that the posterior intensity at time *k-1* is a Gaussian mixture of the form [14]

$$D_{k-1|k-1}(x) = \sum_{i=1}^{J(k-1)} w^i{}_{k-1} N[x; m^i{}_{k-1}, P^i{}_{k-1}] \tag{11}$$

Then the predicted intensity to time *k* is also a Gaussian mixture, and is given by

$$D_{k|k-1}(x) = D_{S\,k|k-1}(x) + D_{\beta\,k|k-1}(x) + \gamma_k(x) \tag{12}$$

where $D_{S\,k|k-1}(x)$ is the PHD of existing targets, $D_{\beta\,k|k-1}(x)$ is the PHD for spawned targets, and $\gamma_k(x)$ is the PHD of spontaneous birth targets.

The density for existing targets, $D_{S\,k|k-1}(x)$, is determined from the linear Gaussian model using the Kalman prediction equations,

 ISBN 978-86-7466-930-3

$$D_{S\,k|k-1}(x) = p_{S\,k} \sum_{j=1}^{J(k-1)} w^{j}{}_{k-1} N[x; m^{j}_{S\,k|k-1}, P^{j}_{S\ k|k-1}] \quad (13)$$

where

$$m^{j}_{S\,k|k-1} = F_{k-1} m^{j}{}_{k-1} \quad (14)$$

$$P^{j}_{S\ k|k-1} = Q_{k-1} + F_{k-1} P^{j}{}_{k-1} F^{T}{}_{k-1} \quad (15)$$

and similarly for the spawned target density,

$$D_{\beta\,k|k-1}(x) = \sum_{j=1}^{J(k-1)} \sum_{l=1}^{J_{\beta}(k)} w^{j}{}_{k-1} w^{l}_{\beta} N[x; m^{j,l}_{\beta\ k|k-1} P^{j,l}_{\beta\ k|k-1}] \quad (16)$$

where

$$m^{j,l}_{\beta\ k|k-1} = F^{l}_{\beta\,k-1} m^{j}{}_{k-1} + d^{l}_{\beta\,k-1} \quad (17)$$

$$P^{j,l}_{\beta\ k|k-1} = Q^{l}_{\beta\,k-1} + F^{l}_{\beta\,k-1} P^{j}{}_{k-1} (F^{l}_{\beta\,k-1})^{T} \quad (18)$$

2 Update step

Under the above assumptions, and that the predicted intensity to time *t* is a Gaussian mixture of the form [15]

$$D_{k|k-1}(x) = \sum_{i=1}^{J(k|k-1)} w^{i}{}_{k|k-1} N[x; m^{i}{}_{k|k-1}, P^{i}{}_{k|k-1}] \quad (19)$$

Then the posterior intensity at time k is also a Gaussian mixture, and is given by

$$D_{k|k}(x) = [1 - p_{D\,k}] D_{k|k-1,}(x) + \sum_{y \in Y^{k}} \sum_{j=1}^{J(k|k-1)} w^{j}{}_{k}(y) \quad N[x; m^{j}{}_{k|k}(y), P^{j}{}_{k|k}] \quad (20)$$

where the weights are calculated according to the closed form PHD update equation,

$$w^{j}{}_{k}(y) = \frac{p_{D\,k} w^{i}{}_{k|k-1} N[y; H_{k} m^{j}{}_{k|k-1}, R_{k} + H_{k} P^{j}{}_{k|k-1} H^{T}{}_{k}]}{K_{k}(y) + p_{D\,k}) \sum_{l=1}^{J(k|k-1)} w^{l}{}_{k|k-1} N[y; H_{k} m^{l}{}_{k|k-1}, R_{k} + H_{k} P^{l}{}_{k|k-1} H^{T}{}_{k}]} \quad (21)$$

and the mean and covariance are updated with the Kalman filter update equations,

$$m^{j}{}_{k|k}(y) = m^{j}{}_{k|k-1}(y) + K^{j}{}_{k}[y - H_{k} m^{j}{}_{k|k-1}(y)] \quad (22)$$

$$P^{j}{}_{k|k} = [I - K^{j}{}_{k} H_{k}] P^{j}{}_{k|k-1} \quad (23)$$

$$K^{j}{}_{k} = P^{j}{}_{k|k-1} H^{T}{}_{k} [H_{k} P^{j}{}_{k|k-1} H^{T}{}_{k} + R_{k}]^{-1} \quad (24)$$

B. GMPHD practical implementation of algorithm

At begin, we given weights, mean and covariance of the each track:

$$\{w^{(i)}{}_{k-1}, m^{(i)}{}_{k-1}, P^{(i)}{}_{k-1}\}_{i=1}^{J(k-1)}$$

At each scan we given measurement set $Z(k)$, from the radar sensor .

Step 1: **Prediction for birth targets**

$i = 0.$

for j=1,…, $J_{\gamma}(k)$

i=i+1.

$$w^{(i)}{}_{k|k-1} = w^{(j)}_{\gamma\ k},$$

$$m^{(i)}{}_{k|k-1} = m^{(j)}_{\gamma\ k}$$

$$P^{(i)}{}_{k|k-1} = P^{(j)}_{\gamma\ k}$$

End

for j=1,…, $J_{\beta}(k)$

for l=1,…, $J(k-1)$

i=i+1.

$$w^{(i)}{}_{k|k-1} = w^{(l)}{}_{k-1} w^{(j)}_{\beta\ k}$$

$$m^{(i)}{}_{k|k-1} = d^{(j)}_{\beta\ k-1} + F^{(j)}_{\beta\ k-1} m^{(l)}{}_{k-1}$$

$$P^{(i)}{}_{k|k-1} = Q^{(j)}_{\beta\ k-1} + F^{(j)}_{\beta\ k-1} P^{(l)}{}_{k-1} (F^{(j)}_{\beta\ k-1})^{T}$$

end

end

Step 2: **Prediction for existing targets**

for j=1,…, $J(k-1)$

i=i+1.

$$w^{(i)}{}_{k|k-1} = p_{S\,k} w^{(j)}_{\gamma\ k-1},$$

$$m^{(i)}{}_{k|k-1} = F_{k-1} m_{\gamma}^{(j)}{}_{k-1}$$

$$P^{(i)}{}_{k|k-1} = Q_{k-1} + F_{k-1} P^{(j)}{}_{k-1} (F_{k-1})^T$$

end

$$J(k|k-1) = i$$

Step 3: **Construction of PHD update components**

for j=1,…, $J(k|k-1)$

$$\eta^{(j)}{}_{k|k-1} = H_k m^{(j)}{}_{k|k-1}$$

$$S^{(j)}{}_k = R_k + H_k P^{(j)}{}_{k|k-1} H^T{}_k$$

$$K_k = P^{(j)}{}_{k|k-1} H^T{}_k [S^{(j)}{}_k]^{-1}$$

$$P^{(j)}{}_{k|k} = [I - K^{(j)}{}_k H_k] P^{(j)}{}_{k|k-1}$$

end

Step 3: **(Update)**

for j=1,…, $J(k|k-1)$

$$w^{(j)}{}_k = (1 - p_{D,k}) w^{(j)}{}_{k|k-1},$$

$$m^{(j)}{}_k = m^{(j)}{}_{k|k-1},$$

$$P^{(j)}{}_k = P^{(j)}{}_{k|k-1}$$

end

l=0.

for each $y \in Y^k$

l=l+1,

for j=1,…, $J(k|k-1)$

$$w^{(lJ(k|k-1)+j)}{}_k =$$

$$p_{Dk} w^{(j)}{}_{k|k-1} N(y; \eta^{(j)}{}_{k|k-1}, S^{(j)}{}_k)$$

$$m^{[lJ(k|k-1)+j]}{}_k =$$

$$= m^{(j)}{}_{k|k-1} + K^{(j)}{}_k [y - \eta^{(j)}{}_{k|k-1}]$$

$$P^{[lJ(k|k-1)]}{}_k = P^{(j)}{}_{k|k}$$

End

$$w^{[lJ(k|k-1)+j]}{}_k = \frac{w^{[lJ(k|k-1)+j]}{}_k}{\mathrm{K}^{(y)}{}_k + \sum_{i=1}^{J(k|k-1)} w^{[lJ(k|k-1)+i]}{}_k}, \; j = 1, \ldots, J(k|k-1)$$

end

$$J(k) = lJ(k|k-1) + J(k|k-1)$$

Output $\{w^{(i)}{}_k, m^{(i)}{}_k, P^{(i)}{}_k\}_{i=1}^{J(k)}$

C. *Output Calculation*

Finally, we can calculate the output state estimate and covariance (for output purpose only):

$$\hat{x}_{k|k} = \sum_{l=1}^{Np} w_k^l x_k^l \tag{25}$$

$$P_k = \left(\sum_{l=1}^{Np} w_k^l \cdot x_k^l \cdot x_k^{l\,T} \right) - \hat{x}_k \cdot \hat{x}_k^{\,T} \tag{26}$$

IV. Results of experiment

For the purpose of simulations, we propose a two-dimensional scenario (Fig. 1) with an unknown crossing and time varying number of targets in clutter over the region [−500; 500] × [−500; 500]. The state $\boldsymbol{x}_k = [\, x_k \; v_{xk} \; y_k \; v_{yk} \,]^T$, of each target consists of position (x_k; y_k) and velocity (v_{xk}; v_{yk}), while the measurement is a noisy version of the position [16].

Each target has survival probability $p_{S;k}$ = 0,9, detection probability $p_{D;k}$=0,99 and follows the linear Gaussian dynamics. Each simulation experiment consists of a number of simulation runs. In each simulation run, targets will repeat their trajectories. The measurements are generated independently. Each algorithm uses the same set of measurements. False tracks may be initiated using target measurements, either in a conjunction with a clutter measurement, or by using measurements from different targets in different scans. The sampling period of radar sensor is *T=1s*. Duration of the scenario is 70 scans. The implemented GM-PHD is evaluated by Monte Carlo (MC) simulations over representative 2-dimensional test scenario. A target motion scenario (Fig.2) includes non-maneuvering flights modes. Dimension of terrain surveillance is x=500m and y=500m.

Transition matrix and process noise matrix are given by:

$$F = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad Q_k = q \begin{bmatrix} T^3/3 & T^2/2 & 0 & 0 \\ T^2/2 & T & 0 & 0 \\ 0 & 0 & T^3/3 & T^2/2 \\ 0 & 0 & T^2/2 & T \end{bmatrix} \tag{27}$$

respectively. Measurements matrix and measurements noise matrix is given by:

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad R_k = \begin{bmatrix} \sigma_x^2 & 0 \\ 0 & \sigma_y^2 \end{bmatrix} \tag{28}$$

respectively.
For the purpose of performance testing tracking targets, we propose Wasserstein distance diagram.

A. Wasserstein distance:

The Wasserstein distance from theoretical statistics was adopted as a means of defining a performance metric for multi target distances which penalizes when the estimate of the number of targets is incorrect. When the number of targets is estimated correctly, the Wasserstein distance is the same as the Hausdor distance but the Hausdor does not penalise for incorrectly estimating the number of targets. This metric has been used for assessing the performance of the PHD filter.

Let X_t and Y_t be the finite sets of target states and estimated target states at time interval k.. The L^P Wasserstein distance between the two sets is defined as:

$$d_P^W(X_t, Y_t) = \inf_C \Big(\sum_{x_t \in X_t} \sum_{\hat{x}_j \in \hat{X}_t} C_{ij} d(x_i, \hat{x}_j)^P \Big)^{1/P} \tag{29}$$

where C is an $|X_t| \times |\hat{X}_t|$ matrix $\{C_{ij}\}$ such that $\forall i = 1 \dots |X_t|, j = 1 \dots |\hat{X}_t|$:

$$\sum_{i=1}^{|X_t|} C_{ij} = \frac{1}{|\hat{X}_t|}, \sum C_{ij} = \frac{1}{|X_t|}, C_{ij} \geq 0 \tag{30}$$

The L^∞ Wasserstein distance is defined as:

$$d_\infty^W(X_t, \hat{X}_t) = \inf_C \max_{xi \in X_t, \hat{x}_j \in \hat{X}_t} C_{ij} d(x_i, \hat{x}_j) \tag{31}$$

Where $\tilde{C}_{ij} = 1$ if $C_{ij} > 0$ and $\tilde{C}_{ij} = 0$ if $C_{ij} = 0$

B. Two point differencing initializations

Initialization with the difference of successive observations (Two point differentiation initializing) uses measurements located in the 'window' of the trace from two successive scans to initialize the trace. This procedure is repeated for all measurements from the scan k-1 . Consider such a measurement $z_{k-1,j}$. The new trace is initialized by measurement $z_{k-1,j}$ and each selected measurement $z_{k,i}$, forming a Gaussian probability density function with the mean value given as:

$$\hat{x}_k^{(2)} = \begin{bmatrix} z_{k,i} \\ \dfrac{z_{k,i} - z_{k-1,j}}{\Delta T_k} \end{bmatrix} \tag{32}$$

where ΔT_k one is the period of rotation of the radar antenna. Since there is no a priori knowledge of the target speed, it can be modeled through a uniform distribution of a priori pdf measurements. At the moment k, the Np particles of the mean value $\hat{x}_k^{(2)}$ are initialized, as well as the symmetric and semi-finite covariance matrices $P_{(2|2)}$ which correspond to the normal distribution of several variables (Figure 1):

$$x_k^p = N[\hat{x}_k^{(2)}; P_{(2|2)}] \tag{33}$$

where the initial covariance error of the condition is calculated under the assumption that there is no process noise [24]:

$$P_{(2|2)} = \begin{bmatrix} R_k & R_k / T \\ R_k / T & 2R_k / T^2 \end{bmatrix} \tag{34}$$

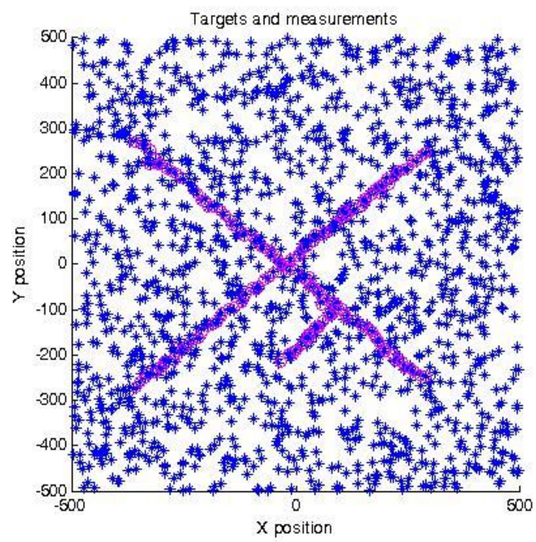

Fig. 1. Simulation scenario: targets and measurements .

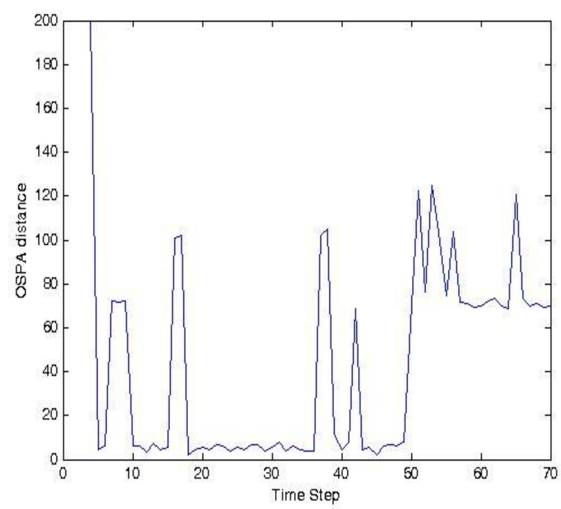

Fig. 2. Wasserstein distance metric (OSPA diagram)

 ISBN 978-86-7466-930-3

Simulation results (OSPA diagram- Fig. 2) show good performance for tracking two crossing targets with one spawned targets, from scan 50 (Fig. 1). Blue dots (measurements) and magenta circle (targets) show good initializing and tracking targets in heavy clutter environments.

V. CONCLUSION

An improved GM PHD algorithm for radar sensor approaches has been presented for tracking multiple targets in high clutter density which has the ability to estimate the number of targets, track the trajectories of the targets over time, operate with missed detections and give the trajectories of the targets in the past once a target has been identified. It has been shown to outperform the ability to operate in clutter with fewer false tracks and can initiate and eliminate targets more accurately.

Future research should better determine the association of radar received measurements with existing targets as well as automatic initialization of targets. The theoretical constraints of the proposed tracking algorithm have been discussed in the case of crossing targets. It is anticipated that the problem of retaining the correct target identity in this scenario can be resolved by considering the previous trajectories of targets.

Especially, the proposed algorithm will be tested in practice on the example of video tracking [17, 18].

REFERENCES

[1] S. Blackman, Multiple Target Tracking with Radar Applications. Artech House, Norwood, 1986.

[2] Challa S., Evans R., Morelande M., Mušicki D.: Fundamentals of Object Tracking, Cambridge University Press 2011

[3] R. Mahler, “Multitarget Bayes filtering via first-order multitarget moments,” IEEE Trans. Aerosp. Electron. Syst., vol. 39, no. 4, pp. 1152–1178, Oct. 2003.

[4] O. Erdinc, P. Willet, and Y. Bar-Shalom. Probability Hypothesis Density Filter for Multitarget Multisensor Tracking. Proc. FUSION 2005.

[5] B. Vo and W. K. Ma. The Gaussian Mixture Probability Hypothesis Density Filter. IEEE Transactions on Signal Processing, 2006.

[6] B. Vo, S. Singh, and A. Doucet, “Sequential Monte Carlo methods for multi target filtering with random finite sets,” IEEE Trans. Aerosp. Electron.Syst., vol. 41, no. 4, pp. 1224–1245, Oct. 2005.

[7] D. Clark and J. Bell, “Convergence results for the particle PHD filter,” IEEE Trans. Signal Process., vol. 54, no. 7, pp. 2652–2661, Jul. 2006.

[8] R. Mahler. Multi-target Bayes filtering via first order multi target moments. IEEE Transactions on Aerospace and Electronic Systems, 39, No.4:1152-1178, 2003.

[9] K. Panta, B. Vo, S. Singh, and A. Doucet. Probability hypothesis density filter versus multiple hypothesis tracking. Proceedings of SPIE, Volume 5429 Signal Processing, Sensor Fusion, and Target Recognition XIII, Ivan Kadar, Editor, August 2004, pp. 284-295.

[10] Z. Radosavljević, T.L. Song, B.Kovacevic, Linear Multi-Target IPF Algorithm for Automatic Tracking, Scientific Technical Review, 2016,Vol.66,No.1,pp.3-10, Belgrade 2016.

[11] Z. Radosavljević, D. Mušicki, B. Kovačcević, W. C. Kim, and T. L. Song, Integrated particle filter for target tracking in clutter, IET proceedings on Radar Sonar and Navigation, DOI: 10.1049/iet-rsn.2014.0341 , Print ISSN 1751-8784, May, 2015.

[12] T. L. Song, D. Mušicki, D.S. Kim and Z.Radosavljević, Gaussian mixtures in multi-target tracking: a look at Gaussian mixture probability hypothesis density and integrated track splitting, IET proceedings on Radar Sonar and Navigation, Vol 6, no 5, pp. 359-364, June 2012.

[13] B. Kovačević, Z. Radosavljević and D. Ivković, "The Interacting Multiple Model GM PHD for Single Target Tracking", Proc. of 64. International Conference IcETRAN 2020, Belgrade, September 28.-29., 2020.

[14] Z. Radosavljevic,D. Musicki, B. Kovacevic, W.C. Kim and T. L. Song, ‘Integrated Particle Filter for Target Tracking’, Proceedings of International Conference on Electronics, Information and Communications (ICEIC). Kota Kinabalu, Malesya, Januar 2014.

[15] Z. Radosavljevic, D. Paunovic, D. Ivkovic, D. Nikolic, An Efficient Solid State VHF Band Radar Transimitter,, 6th International Scientific Conference on Defensive Technologies OTEH 2014, Belgrade, 2014.

[16] Z. Radosavljevic, D. Musicki, Limits of target tracking in heavy clutter, ASIA-Pacific International Conference of Synthetic Aperture Radar APSAR 2011, Seoul, Republic of Korea., 2011.

[17] Z. Fu, F. Angelini, J. Chambers, and S. M. Naqvi, “Multi-Level Cooperative Fusion of GM-PHD Filters for Online Multiple Human Tracking,” IEEE trans. Multimedia. (Early Access), Mar. 2019.

[18] N. L. Baisa and A. Wallace, “Development of a n-type GM-PHD filter for multiple target, multiple type visual tracking.” J. Vis. Commun. Image Represent., vol. 59, pp. 257–271, 2019