Damper Winding Inductances Calculation by Winding Function Approach

Aldin Kajević, Gojko Joksimović, *Senior Member, IEEE*

***Abstract*—The paper presents a procedure for dq model parameters calculation of a synchronous turbogenerator using winding function theory, with special emphasis on the damper winding. The advantage of this procedure is that the real spatial distribution of all windings in the machine is taken into account, therefore, taking into account all spatial harmonics simultaneously. A real synchronous turbogenerator of type TBB-200-2A was analyzed as a case study.**

***Index Terms*—Damper winding, Inductance, Synchronous machine, Turbo-generator, Winding function.**

I. Introduction

Mathematical model of synchronous machine projected on two mutually orthogonal axes, *dq* model, is the most common dynamic model of synchronous machine, [1], [2]. Although the model itself is often found in the literature, in rare cases attention is paid to determining the parameters that appear in the model itself. Additionally, model parameters are most often given in unit values. The key parameters of the model that are at the same time the most demanding to determine are the self and mutual inductances of the windings. Analytical expressions that assume an ideal, sinusoidal spatial distribution of windings are often used when determining these parameters, or, which is the same, only the fundamental harmonic of the real spatial winding distribution is taken into account, [3], [4]. In this paper, using the theory of the winding functions, [5], [6], [7], [8], all spatial harmonics of the magnetomotive force (mmf) of the real winding are taken into account simultaneously.

The generic synchronous machine is characterized by the existence of three symmetrical armature phase windings as well as the field and damper winding on the rotor.

The paper presents a methodology that allows direct and fairly intuitive determination of self and mutual inductances of windings, inductances that appear as parameters of the *dq* model of the machine. The first step in order to calculate inductances is to determine the winding functions of the stator windings and the damper windings of the machine in the natural frame of reference. However, the following convention should be introduced here: in order to avoid rugged constructions as "winding function of field winding" or "winding function of stator phase winding", etc. a simpler sentence construction will be used – simply "field winding function" or "damper winding function" or "stator phase winding function". When the winding functions are known, then in the case of stator phase windings, their equivalent winding functions are determined in a rotating *dqn* reference frame fixed to the rotor. The role of these functions is twofold. Firstly, they enable obtaining self and mutual inductances of the stator windings directly in the *dqn* system. Secondly, their existence enables, in combination with the damper winding functions along the *d* and *q* axes, the direct calculation of its mutual inductances. The procedure for obtaining the damper winding functions along the *d* and *q* axes is also presented in this paper and these functions represent one of the main results of this paper.

Aldin Kajević is with the Faculty of Electrical Engineering, University of Montenegro, Cetinjski put b.b, 81000 Podgorica, Montenegro (e-mail: Aldin.Kajevic@ucg.ac.me)

Gojko Joksimović is with the Faculty of Electrical Engineering, University of Montenegro, Cetinjski put b.b, 81000 Podgorica, Montenegro (e-mail: Gojko.Joksimovic@ucg.ac.me), (https://orcid.org/0000-0002-2764-1540)

The whole procedure is illustrated on the example of a synchronous turbogenerator of type TBB-200-2A. As a final result, all self and mutual inductances that appears as the parameters of the *dq* model are determined. A comparison of the obtained parameters concerning the stator windings with the factory data of the generator was performed and a high degree of agreement was observed.

II. DQ Model of Synchronous Machine

The voltage equations of a synchronous turbogenerator in matrix form are given below, [1]. The indexes *a*, *b* and *c* are associated with the values concerning the three phase windings of the stator, in the natural (*abc*) frame of reference. The indexes *d* and *q* are associated with the damper winding along the *d* and *q* axes, respectively. Index *f* refers to the field winding. The indexes *s* and *r* refer to the stator and rotor, respectively.

$$\left[U_{abcs}\right]=\left[R_s\right]\left[i_{abcs}\right]+\frac{d}{dt}\left[\Psi_{abcs}\right] \tag{1}$$

$$\left[U_{dqfr}\right]=\left[R_r\right]\left[i_{dqfr}\right]+\frac{d}{dt}\left[\Psi_{dqfr}\right] \tag{2}$$

The equations of magnetic coupling for the stator and rotor windings, in matrix form, are:

$$\left[\Psi_{abcs}\right]=\left[L_{abcs}\right]\left[i_{abcs}\right]+\left[L_{abcsr}\right]\left[i_{dqfr}\right] \tag{3}$$

 ISBN 978-86-7466-930-3

$$\left[\psi_{dqfr}\right]=\left[L_{abcsr}\right]^{T}\left[i_{abcs}\right]+\left[L_{dqfr}\right]\left[i_{dqfr}\right] \quad (4)$$

Vectors of voltages, currents and fluxes that appear in the previous expressions together with the matrices of inductance and resistance are given in the Appendix. The same will be done with the vectors and matrices in the continuation of the work.

It is now necessary to transform the previous equations from the natural (*abc*) frame of reference to the rotating *dqn* coordinate system. The transformation will be done using the transformation matrix [$T(\theta)$]. The angle θ in the transformation matrix is the angle that the rotating reference frame, fixed to the rotor, occupies at a certain point in time in relation to the reference, i.e. initial position. For a coordinate system that rotates with an arbitrary angular velocity ω, assuming that the initial angle is equal to zero, at time t=0, the following holds:

$$\theta=\omega t \quad (5)$$

After the transformation of equations, procedure of which will not be presented in detail here, the following equations are obtained in the *dqn* system,

$$\left[U_{dqns}\right]=\left[R_s\right]\left[i_{dqns}\right]+\frac{d\left(\left[\psi_{dqns}\right]\right)}{dt}+\left[\omega\mathrm{x}\right]\left[\psi_{dqns}\right] \quad (6)$$

$$\left[U_{dqfr}\right]=\left[R_r\right]\left[i_{dqfr}\right]+\frac{d}{dt}\left[\psi_{dqfr}\right] \quad (7)$$

$$\left[\psi_{dqns}\right]=\left[L_{dqns}\right]\left[i_{dqns}\right]+\left[L_{dqnsr}\right]\left[i_{dqfr}\right] \quad (8)$$

$$\left[\psi_{dqfr}\right]=\frac{3}{2}\left[L_{dqnsr}\right]^{T}\left[i_{dqns}\right]+\left[L_{dqfr}\right]\left[i_{dqfr}\right] \quad (9)$$

where:

$$\left[L_{dqns}\right]=\left[T(\theta)\right]\left[L_{abcs}\right]\left[T(\theta)\right]^{-1} \quad (10)$$

$$\left[L_{dqnsr}\right]=\left[T(\theta)\right]\left[L_{abcsr}\right] \quad (11)$$

III. Winding functions in DQ system, inductances and method of their determination

The stator winding functions in the *dqn* system are given in expression (12). There $N_a(\theta_m)$, $N_b(\theta_m)$ and $N_c(\theta_m)$ are the stator phase windings functions in the natural frame of reference, and θ_m is the mechanical angle.

In addition to the previously listed winding functions, it is necessary to determine the function of the field winding $N_{fr}(\theta_m)$, as well as the damper winding functions along the *d* and *q* axes, $N_{dr}(\theta_m)$ and $N_{qr}(\theta_m)$, respectively. All these functions will be determined in the following by applying the winding function theory on the example of a real synchronous turbogenerator of type TBB-200-2A.

When the winding functions for all windings are determined then self and mutual inductances in the model matrices can be obtained by numerical integration. Some of the inductances in the matrices [L_{dqns}], [L_{dqnsr}] and [L_{dqfr}] are given by expressions (15) - (20), just for illustration purposes.

$$N_{ds}=\frac{2}{3}\left(N_a(\theta_m)\sin(\theta)+N_b(\theta_m)\sin\left(\theta-\frac{2\pi}{3}\right)+N_c(\theta_m)\sin\left(\theta+\frac{2\pi}{3}\right)\right) \quad (12)$$

$$N_{qs}=\frac{2}{3}\left(N_a(\theta_m)\cos(\theta)+N_b(\theta_m)\cos\left(\theta-\frac{2\pi}{3}\right)+N_c(\theta_m)\cos\left(\theta+\frac{2\pi}{3}\right)\right) \quad (13)$$

$$N_{ns}=\frac{\sqrt{2}}{3}\left(N_a(\theta_m)+N_b(\theta_m)+N_c(\theta_m)\right) \quad (14)$$

$$L_{dsds}=\frac{3\mu_0 rl}{2g}\int_0^{2\pi}N_{ds}^2(\theta_m,\theta)d\theta_m \quad (15)$$

$$L_{dsqs}=\frac{3\mu_0 rl}{2g}\int_0^{2\pi}N_{ds}(\theta_m,\theta)N_{qs}(\theta_m,\theta)d\theta_m \quad (16)$$

$$L_{dsdr}=\frac{\mu_0 rl}{g}\int_0^{2\pi}N_{ds}(\theta_m,\theta)N_{dr}(\theta_m,\theta_r)d\theta_m \quad (17)$$

$$L_{dsqr}=\frac{\mu_0 rl}{g}\int_0^{2\pi}N_{ds}(\theta_m,\theta)N_{qr}(\theta_m,\theta_r)d\theta_m \quad (18)$$

$$L_{drdr}=\frac{\mu_0 rl}{g}\int_0^{2\pi}N_{dr}^2(\theta_m,\theta)d\theta_m \quad (19)$$

$$L_{drqr}=\frac{\mu_0 rl}{g}\int_0^{2\pi}N_{dr}(\theta_m,\theta)N_{qr}(\theta_m,\theta_r)d\theta_m \quad (20)$$

etc.

IV. Stator phase winding and field winding functions

The synchronous generator of type TBB-200-2A is a turbogenerator that is cooled by water and hydrogen. Its rated active power is 200 MW, rated voltage is 15.75 kV, rated frequency 50 Hz. It is a two-pole generator. On the stator of this generator, there exists 60 slots in which the stator winding is manufactured. The stator winding consists of three symmetrical phase windings. Each phase winding additionally consists of two half-windings connected in parallel. The phase windings are connected in star conection. Due to the existence of two parallel connected half-windings, this conection is also known as a double star conection. The three-phase stator winding consists of sixty coil groups with one coil, or twenty coils per phase - ten per half-winding. In Fig. 1 all slots and all coil groups on the stator and rotor are shown and numbered. On the stator, the shades of red represent the conductors of the phase A winding. More intense red color shows the first and lighter one the second half-winding. The other two phases are represented by shades of yellow and green - phase B in yellow and phase C in green. All slots, coil

 ISBN 978-86-7466-930-3

groups are clearly numbered and the adopted reference directions of currents and mechanical angle are shown. Based on the previous data, the functions of the phase windings can be determined, [5]-[8]. Thus, the winding function of phase A is given in Fig. 2.

Field and damper winding are mounted in 36 rotor slots. The rotor slot pitch is $\tau_r=2\pi/52$. The field winding consists of 18 coil groups, and each coil group consists of seven coils. Coil groups of the field winding are marked in green and red on the rotor, Fig. 1. Damper winding consists of conductive wedges made of aluminum alloy which close the slots in which the field winding is located. These bars are short-circuited on the front and back sides by a conductive tension rings. They are shown in blue in Fig. 1 The waveform of the field winding function is given on Fig. 3 – that is at the same time the waveform of the field winding mmf for a unit current, [1].

The functions of the fictive stator windings in the rotating frame of reference fixed to the rotor are obtained from the original functions of the phase windings using expresions (12)-(14). The winding functions obtained in this way, under the *d* and *q* axes, are given in Fig. 4 for the case $\theta=0^{o}$.

All of these functions will be used in the following paragraphs to calculate the inductances that appear in the *dq* model.

The function of the field winding that is located on the rotor does not need to be reduced because the rotating frame of reference is already fixed to the rotor. The magnetic axis of this winding is the *d* axis, Fig. 1.

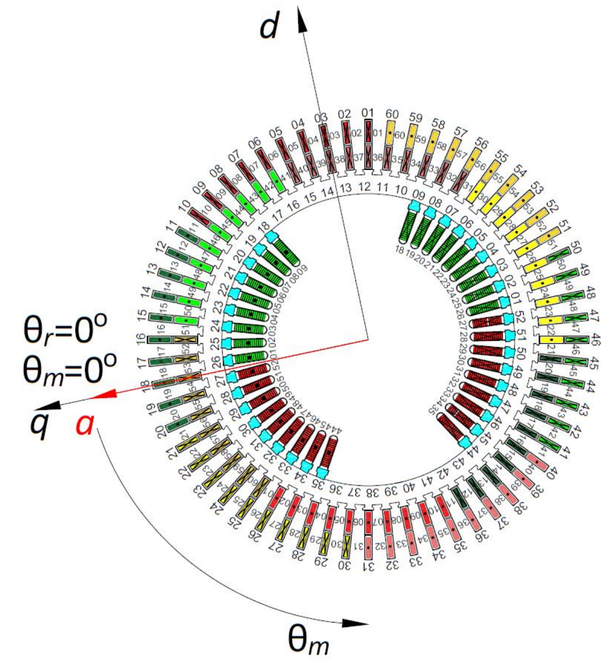

Fig.1. Design of phase, field and damper windings of the TBB-200-2A turbogenerator

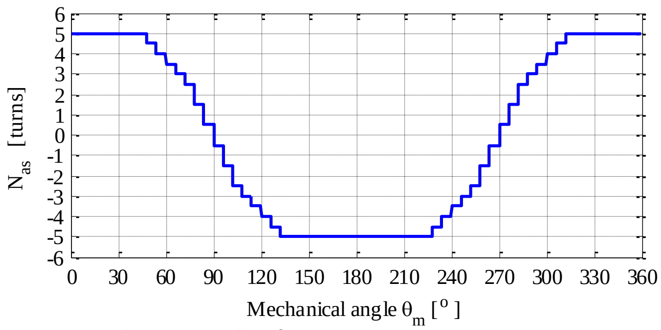

Fig.2. Stator phase A - winding function

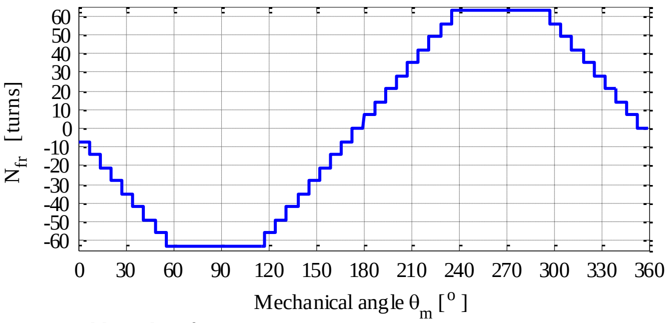

Fig.3. Field winding function

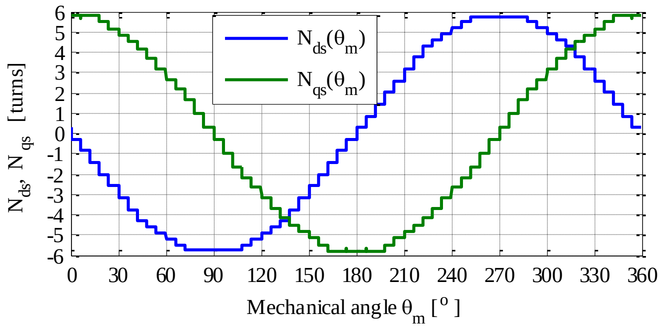

Fig.4. Fictive stator winding functions reduced to a rotating frame of reference, $\theta=0^{o}$

V. Damper winding functions

Structurally, the damper winding consists of conductive bars located in the top of the rotor slots. These bars are short-circuited on the front and back sides by the conductive tension rings. From the construction of the damper winding itself, it is clear that the same currents do not flow in short-circuited conducting bars, which, at first glance, makes this winding complicated for analysis using the concept of winding function. Namely, in order to be able to apply the definition of the winding function, it is necessary that one and the same current flows through that winding, [1]. This condition is met for armature phase windings and the field winding. This difficulty in determining the function of the damping winding can be overcome by taking into account some assumptions and neglections. As the bars on the rotor are short-circuited, currents in them are induced as a result of time-varying magnetic flux generated by currents in the phase windings of the stator. Due to the symmetry of the phase windings, it is sufficient to observe the case of induced currents resulting from the magnetic flux generated by one of the armature

 ISBN 978-86-7466-930-3

phase windings. The phase A winding will be observed here. Based on the currents induced in the bars as a consequence of the current in the phase winding, a usable damper winding function along the *d* and *q* axes can be obtained.

In order to determine the function of the damper winding along the *d* axis, Fig. 5 shows the developed scheme of the damper winding for the case when the *d* axis is in the same position as the axis of the phase A winding, i.e. the *d* axis is below the maximum value of fundamental harmonic of winding A mmf wave.

The N_{as} function shown in red in Fig. 5 represents the fundamental harmonic of the phase A winding function. Multiplying function $N_{as}(\theta_m)=N_m\cos(\theta_m)$ by the sinusoidal current in the winding, $I_a\sin(\omega_e t)$, gives a pulsating mmf. As the air-gap in the turbogenerator is uniform (ignoring the stator and rotor slots), the magnetic flux density wave created by the phase A winding will also be sinusoidal along the rotor circumference. Due to the existence of symmetry, the induced currents in bars 1 and 1' are the same but of opposite polarity, so these two bars can be considered as one fictious coil. The same situation is with 2-2', 3-3' pair of bars, etc. which are under same pole. The magnetic fluxes coupled by these fictitious coils are given by the following expressions,

$$\phi_{11'} = \frac{2\mu_0 N_m I_a rl \sin(9z/2)}{g}\sin(\omega_e t) \qquad (21)$$

$$\phi_{22'} = \frac{2\mu_0 N_m I_a rl \sin(11z/2)}{g}\sin(\omega_e t) \qquad (22)$$

etc.

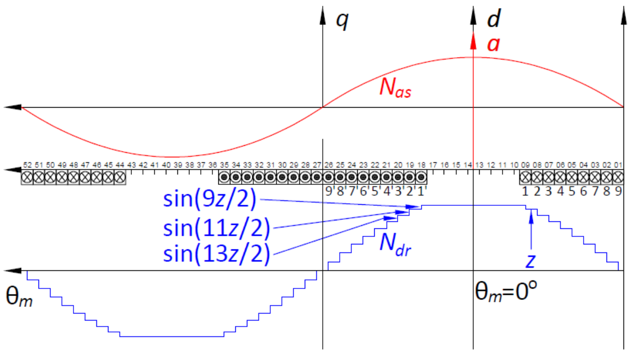

Fig.5. Damper winding function along the *d* axes

What can be seen from the previous expressions is the fact that the fluxes in the fictitious coils differ only by the factors sin(9*z*/2), sin(11*z*/2), ... This difference is due to the difference in the area occupied by these coils, or differences in coil step. The induced electromotive forces in them differ for the same factors. The resistance and leakage inductance of each bar is the same. The resistance and leakage inductance of the ring segments connecting the two bars of the fictitious coil is different for each coil due to the difference in length. However, as the resistance and leakage inductance of the ring segments are usually very small, [1], around thousend times smaller than values for bars, they will not be considered here. As a consequence of the previously mentioned neglection, the currents in the fictitious windings will be the ratio of the induced electromotive force and the double impedance of the bar. Based on the above, it is concluded that the currents will differ only by the factors sin(9*z*/2), sin(11*z*/2), ..., too. Now the damper winding can be viewed as a fictive, concentrated winding through whose coils flows the same current. However, each coil has a number of conductors that differs by a factor of sin(9*z*/2), sin(11*z*/2), ... etc. In this way the mmf of the damper winding remains unchanged. The damper winding function obtained in this way is sketched in Fig. 5, in blue. Fig. 7 shows the exact shape of the damper winding function along the *d* axis, for the analysed machine, taking the position of the *q* axis as the starting position for the mechanical angle when it is in the same position as the phase A winding axis, $\theta_r=0^o$.

The procedure for obtaining the damper winding function along the *q* axis is similar to the previous procedure. Now, Fig. 6, *q* axis is placed in a position that coincides with the magnetic axis of the phase A winding. Due to the obvious symmetry the currents induced in bars 1 and 1' are the same in magnitude but opposite in direction, so these two bars can be viewed as a fictive coil. The situation is similar with other pairs of bars 2-2’, 3-3’, etc. The magnetic fluxes coupled by these fictive coils are:

$$\phi_{11'} = \frac{2\mu_0 N_m I_a rl \sin(z/2)}{g}\sin(\omega_e t) \qquad (23)$$

$$\phi_{22'} = \frac{2\mu_0 N_m I_a rl \sin(3z/2)}{g}\sin(\omega_e t) \qquad (24)$$

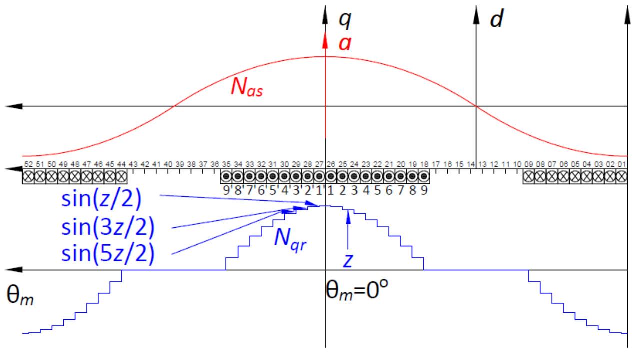

Fig.6. Damper winding function along the *q* axes

Now the factors are sin(*z*/2), sin(3*z*/2), etc. As before, ignoring the impedances of the ring segments that connect the corresponding bars, one can conclude that the currents will differ from each other only by the factors sin(*z*/2), sin(3*z*/2), etc. Now the damper winding along the *q* axis can be considered as fictitious concentrated winding through whose coils the same current flows, but each of the coils has different number of conductors. In this way the mmf of the damper winding remains unchanged. The function of the damper winding along the *q* axis is sketched in Fig. 7 in blue. Fig. 8 gives the exact waveform of the damper winding function along the *q* axis, for the analyzed machine and for the adopted reference directions from Fig. 1.

ISBN 978-86-7466-930-3

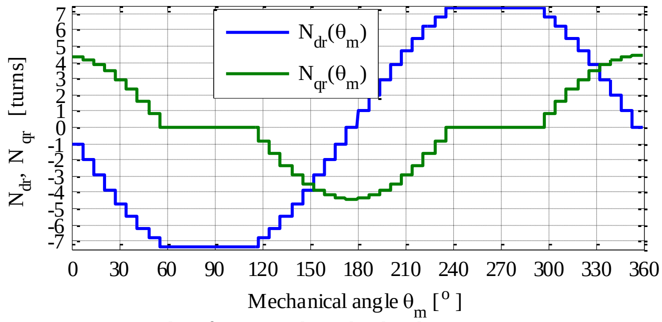

Fig.7. Damper winding functions along the *d* and *q* axes

VI. Results

The winding functions defined in the previous paragraphs enable the determination of all inductances that appear in the equations of the *dq* model. As the rotating frame of reference is fixed to the rotor all the inductances of the model are constant valued. The values of inductances obtained on the example of the analyzed generator are given in Table I.

TABLE I

CALCULATED INDUCTANCES FOR ANALYSED MACHINE

L_{dsds}	L_{qsqs}	L_{dsqs}	L_{dsns}	L_{qsns}	L_{dsdr}	L_{qsqr}	L_{dsqr}
mH	mH	mH	mH	mH	mH	mH	mH
6.6	6.6	0.0017	0.0	0.0	5.9	2.65	0.16
L_{qsdr}	L_{dsfr}	L_{qsfr}	L_{drdr}	L_{qrqr}	L_{drqr}	L_{drfr}	L_{qrfr}
mH	mH	mH	mH	mH	mH	mH	mH
–0.36	48.8	–2.95	8.0	1.8	0.0	65.9	–0.0023

What can be noticed from the previous Table is that there is a certain magnetic coupling between those windings that are located on orthogonal axes (except on the fictitious *n* axis which is normal to the plane formed by the *d* and *q* axes). This magnetic coupling is, however, very weak, as illustrated by the obtained values of the mutual inductances of the windings along these axes: L_{dsqs}, L_{dsqr}, L_{qsdr}, L_{qsfr} and L_{qrfr}. The existence of these mutual inductances is a consequence of higher spatial harmonics which are taken into account by the winding functions.

By adding the leakage inductance of armature phase winding, which for this generator is 0.5 mH to the inductance L_{dsds}, synchronous inductance along the *d* axis is 7.1 mH. Multiplying this value by the electric angular frequency results in a synchronous reactance along the *d* axis of this generator: X_d =2.2305 Ω. By dividing this reactance with the base impedance of the generator, Z_b=1.0556 Ω, unit value of synchronous reactance along the *d* axis can be obtained: x_d=2.113. The unit value of synchronous reactance given in the "passport" of this generator is x_d =2.106. This very good match, testifies to the usefulness of this procedure in determining the parameters of the *dq* model.

VII. Conclusion

The method of determining the parameters of the *dq* model of a synchronous generator using winding function theory is presented in this paper. The procedure enables direct obtaining of parameters without the need for a large number of mathematical manipulations over matrix equations. The method of determining the functions of the damper winding along the *d* and *q* axes, which enables its modeling, is also presented. The procedure is illustrated on a real synchronous turbogenerator of type TBB-200-2A.

Plan for future research is to develop software that will enable obtaining electromagnetic torque, stator, field and damper windings curents in the time domain using the parameters obtained by the explained procedure.

Appendix

$$[U_{abcs}] = [U_{as} \quad U_{bs} \quad U_{cs}]^T$$

$$[U_{dqfr}] = [U_{dr} \quad U_{qr} \quad U_{fr}]^T$$

$$[i_{abcs}] = [i_{as} \quad i_{bs} \quad i_{cs}]^T$$

$$[i_{dqfr}] = [i_{dr} \quad i_{qr} \quad i_{fr}]^T$$

$$[R_s] = \begin{bmatrix} r_s & 0 & 0 \\ 0 & r_s & 0 \\ 0 & 0 & r_s \end{bmatrix}$$

$$[R_r] = \begin{bmatrix} r_{dr} & 0 & 0 \\ 0 & r_{qr} & 0 \\ 0 & 0 & r_{fr} \end{bmatrix}$$

$$[L_{abcs}] = \begin{bmatrix} L_{asas}+L_{ls} & L_{asbs} & L_{ascs} \\ L_{asbs} & L_{bsbs}+L_{ls} & L_{bscs} \\ L_{ascs} & L_{bscs} & L_{cscs}+L_{ls} \end{bmatrix}$$

$$[L_{abcsr}] = \begin{bmatrix} L_{asdr} & L_{asqr} & L_{asfr} \\ L_{bsdr} & L_{bsqr} & L_{bsfr} \\ L_{csdr} & L_{csqr} & L_{csfr} \end{bmatrix}$$

$$[L_{dqfr}] = \begin{bmatrix} L_{drdr} & L_{drqr} & L_{drfr} \\ L_{drqr} & L_{qrqr} & L_{qrfr} \\ L_{drfr} & L_{qrfr} & L_{frfr} \end{bmatrix}$$

$$[T(\theta)] = \frac{2}{3}\begin{bmatrix} \sin(\theta) & \sin\left(\theta-\frac{2\pi}{3}\right) & \sin\left(\theta+\frac{2\pi}{3}\right) \\ \cos(\theta) & \cos\left(\theta-\frac{2\pi}{3}\right) & \cos\left(\theta+\frac{2\pi}{3}\right) \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$[\omega x] = \begin{bmatrix} 0 & -\omega & 0 \\ \omega & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$[U_{dqns}] = [T(\theta)][U_{abcs}]$$

$$[i_{dqns}] = [T(\theta)][i_{abcs}]$$

 ISBN 978-86-7466-930-3

$$\left[\psi_{dqns}\right] = \left[T(\theta)\right]\left[\psi_{abcs}\right]$$

$$\left[L_{dqns}\right] = \begin{bmatrix} L_{dsds} + L_{ls} & L_{dsqs} & L_{dsns} \\ L_{dsqs} & L_{qsqs} + L_{ls} & L_{qsns} \\ L_{dsns} & L_{qsns} & L_{nsns} + L_{ls} \end{bmatrix}$$

$$\left[L_{dqnsr}\right] = \begin{bmatrix} L_{dsdr} & L_{dsqr} & L_{dsfr} \\ L_{qsdr} & L_{qsqr} & L_{qsfr} \\ L_{nsdr} & L_{nsqr} & L_{nsfr} \end{bmatrix}$$

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