Equivalent Electromechanical Model of a Composite Ultrasonic Transducer

Igor Jovanović and Dragan Mančić

Abstract— **This paper presents an original one-dimensional model of a high-power composite ultrasonic transducer with a new structure. The equivalent circuit method is used for a model that can accurately depict the characteristics of the composite ultrasonic transducer and enable its efficient performance evaluation. The proposed model is verified by comparing the modeled dependencies of input electrical impedance vs. frequency with the experimental results. The equivalent circuit developed in this work can facilitate the design and analysis of complex composite transducer structures.**

Index Terms— **High-power ultrasound, Composite ultrasonic transducer, One-dimensional modeling.**

I. Introduction

The piezoelectric ultrasonic transducer is a device that converts desired electrical signals to ultrasonic waves. The applications of high-intensity ultrasonic waves are based on the adequate exploitation of the non-linear effects associated with high amplitudes, such as the radiation pressure, streaming, cavitation, dislocation in solids, etc. [1].

An ultrasonic transducer is a widely used high-power electromechanical transducer for ultrasonic cleaning, ultrasonic liquid processing, and ultrasonic sonochemistry. There are increasingly needed high-power ultrasonic radiators with large amounts of power radiating surfaces. The development of various power ultrasound applications requires ultrasonic transducers with more significant maximum vibration velocity, energy efficiency, and lower temperature rise [2]. Ultrasonic transducer, which consists of piezoceramic and metal rings, has a low resonant frequency (considering the size of the transducer) and a high-quality factor.

Recent research in the field of powerful ultrasound aims to optimize the design of ultrasonic transducers by numerical and analytical modeling methods and the use of precise devices for measuring vibration, mechanical displacement, and stress [3].

The finite element method (FEM) has been a commonly used numerical modeling method to analyze acoustic characteristics of ultrasonic transducers. As a representative work on the use of the FEM, Kagawa and Yambuchi used this method to assess the effect of dimension and material on the resonance frequency of an ultrasonic transducer [4]. In [5], the finite element technique is used for polymer characterization. Wang et al. used FEM to evaluate the output displacement directions of a composite transducer [6]. In addition, FEM is used to evaluate the effect of structural parameters on the output displacement of an ultrasonic transducer [7]. Lin et al. used FEM to evaluate the composite transducer's radial radiation acoustic field distribution [8].

The need for extensive computing resources and long analysis time constitutes the main disadvantage of the FEM [9]. The FEM typically requires a long analysis time and considerable computational resources despite its widespread usage. Therefore, there is a strong need for a more efficient method for analyzing the performance characteristics of the ultrasonic transducer with high accuracy.

The most widely used analytical modeling approach for ultrasonic transducers found in literature is an application of one-dimensional theory using equivalent electromechanical circuits [10]. The equivalent circuit is a method that can analyze the acoustic characteristics of transducers more simply and efficiently than the FEM [9]. It has been utilized to design and analyze various transducers [11].

In their simplest form, ultrasonic transducers are represented by one-dimensional models that represent networks with one electrical and two mechanical approaches. However, when the modeling considers the influence of other parameters (influence of bolt, electrodes, insulators, various electrical connections, prestress, loads, power, etc.) of the transducer, there is an increase in the number of electrical and mechanical approaches in the electromechanical equivalent circuit [10]. Additionally, in the [12], it has been confirmed that using equivalent electromechanical circuits is still possible to model more complex transducer constructions with reasonable accuracy.

Therefore, the composite transducer with a new structure, analyzed in this paper, is presented in the simplest form as a network with two electrical and two mechanical approaches.

II. Analytical One-Dimensional Modelling of Composite Transducer

A new structure of the composite transducer is shown in Fig. 1(a). The composite transducer contains a central mass (2) placed between the two active layers of the transducer ($PZT_{1,2}$ and $PZT_{3,4}$) and two metal endings (1 and 3) connected to the central mass by two central bolts.

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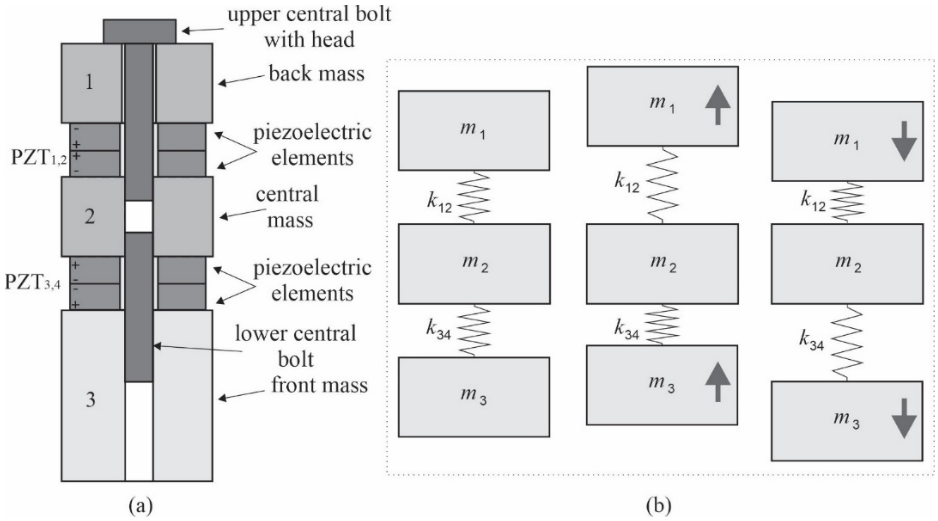

Fig. 1. The composite transducer with a new structure (a), represents of the simplest oscillatory structure of composite transducer (b).

Due to the mutually opposite polarization of the active piezoelectric elements connected to the same power supply, the masses in such a construction oscillates in the manner shown in Fig. 1(b). The three masses constituting this system are m_1, m_2, and m_3 (it is assumed in Fig. 1(b) that the masses are equal to each other), while k_{12} and k_{13} are the stiffness constant.

In its simplest form, the proposed composite transducer is a simple mechanical combination of two half-wave ultrasonic transducers with a sandwich structure that oscillates in the thickness direction (two Langevin-type transducers) [13].

Since the metal endings in the proposed structure are not of the same material, the composite transducer is not bidirectional. The proposed composite transducer has greater flexibility in operation than conventional transducers, which is reflected, among other things, in the possibility of independent excitation of the upper and lower active layer with different signals.

In this paper, modelling of the realized composite transducer with new structure, which represents a special unidirectional composite ultrasonic transducer, is performed. Prestressing the structure is achieved using two central bolts that are in contact with the central mass. The proposed model was adapted based on the structures of the composite transducer and shown as an equivalent electromechanical circuit shown in Fig. 2.

Elements of electromechanical circuits corresponding to isotropic and asymmetric metal parts made of different materials are calculated as:

$$Z_{i1} = jZ_{ci} tg \frac{k_i l_i}{2} \tag{1}$$

$$Z_{i2} = \frac{-jZ_{ci}}{\sin(k_i l_i)} \tag{2}$$

wherein $Z_{ci}=\rho_i v_i P_i$ and $k_i=\omega/v_i$ (for i=1, 2, and 3) are characteristic impedances and the corresponding wave numbers. ρ_i are densities, l_i and P_i are lengths and surface areas of the cross-sections, and v_i are the velocities of longitudinal ultrasonic waves propagation through the corresponding elements.

Elements of the circuit shown in Fig. 2 correspond to the piezoceramic rings in the upper active layer (PZT_{12}), and the piezoceramic rings in the lower active layer (PZT_{34}). These elements are determined as:

$$Z_{p1} = jZ_{cp} tg \frac{nk_p l_p}{2} \tag{3}$$

$$Z_{p2} = \frac{-jZ_{cp}}{\sin(nk_p l_p)} \tag{4}$$

wherein $Z_{cp}=\rho_p v_p P_p$ and $k_p=\omega/v_p$ are characteristic impedances and corresponding wave numbers, respectively. ρ_p, l_p, P_p are densities, lengths, and surface areas of the piezoceramic cross-sections, v_p are velocities of longitudinal ultrasonic waves propagation, respectively. The input electric voltages and currents are marked as V, I_{12}, and I_{34}.

The piezoceramic models consist of capacitance $C_0=n\varepsilon_{33}^S P_p/l_p$, and ideal transformers with transmission ratios $N=h_{33}C_0/n$, wherein n is the number of piezoceramic rings per active layer (n=2). The piezoelectric properties of the transducer active layers are represented by the piezoelectric constant h_{33} and the relative dielectric constant of the pressed ceramic ε_{33}^S.

Piezoceramic rings are mechanically connected in series with central mass, back and front endings. Back and front endings are closed with acoustic impedances Z_R and Z_E, which are in this case negligible because experimental measurements were conducted with unloaded transducers oscillating in the air.

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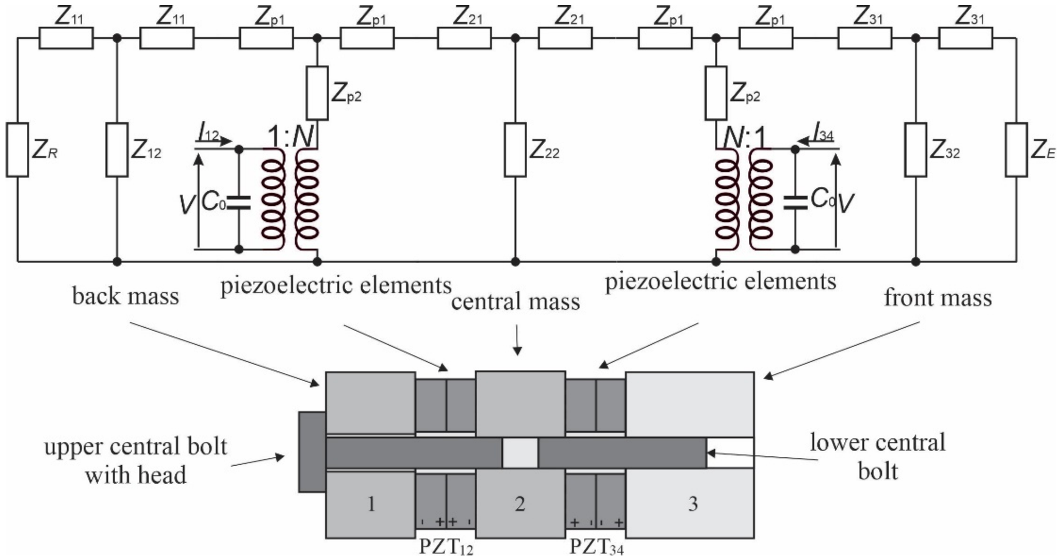

Fig. 2. One-dimensional model of the composite transducer.

Based on Eqs. (1-4) it is obvious that the transducer frequency response depends on the material characteristics of its constituting parts and their geometric dimensions.

In the proposed transducer model, it is assumed that the circuit elements are ideal, i.e. they do not have losses. Losses can be included if piezoelectric constants and constants of elasticity of the transducer metal parts are in the form of complex numbers, in which the imaginary parts represent losses.

III. Simulation and Experimental Results

Table 1 shows the dimensions of the individual composite transducer. Dimensions of the exciting piezoceramic rings are Ø38/Ø13/6.35 mm, and rings are made of PZT8 piezoceramic equivalent material [14]. L_i is the length, a_i and b_i are the outer and inner diameters of the corresponding i-th element. The front ending is made of a dural, while the back ending and the central mass are made of steel with the standard material properties. The electrical impedance measurements are conducted using a Microtest 6366 Precision LCR Meter.

TABLE I
DIMENSIONS OF COMPOSITE TRANSDUCER USED IN EXPERIMENTAL ANALYSIS

Dimension [mm]	Composite transducer
L_1=L_2	11
L_3	37
a_1=a_2=a_3	40
b_1=b_3	9
b_2	8

There is a similarity between the modeled and experimental dependences, as shown in Fig. 2. Since it is a composite transducer with a larger ratio of length and transverse dimensions, the proposed one-dimensional model gives satisfactory results during transducer analysis in the first resonant mode.

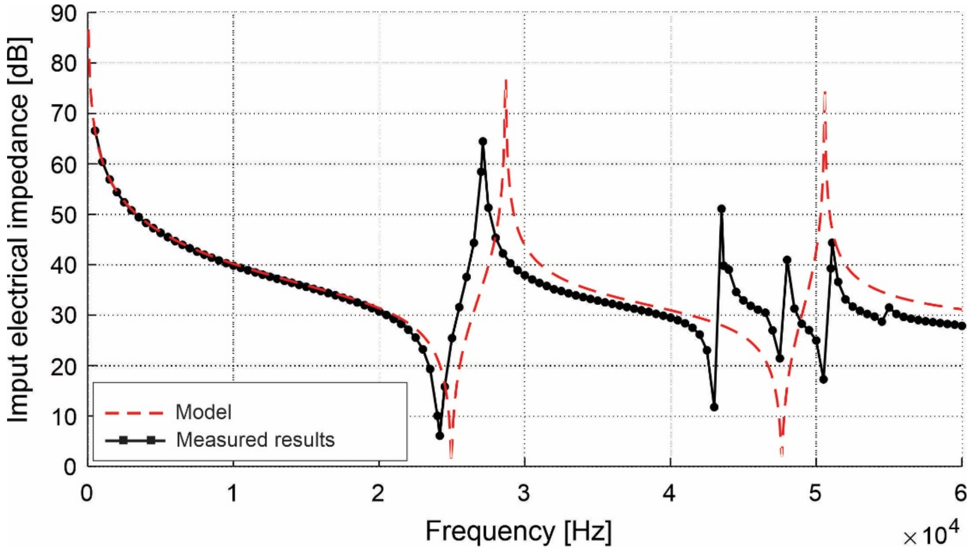

Fig. 3. Input electrical impedance vs. frequency for the proposed composite transducer

The measured resonant frequency of the fundamental resonant mode is 24.16 kHz. The calculated resonant frequency using the proposed model is 24.95 kHz, where the error made by the one-dimensional model in determining this resonant frequency is 3.27%. When it comes to the antiresonant frequency, the measured value is 27.12 kHz, while the model calculated 28.7 kHz, i.e., the error made by the model is 5.83%. The proposed model can predict the general shape of the second resonant mode but with significant error. The measured resonant frequency of the second resonant mode is 43 kHz, while the resonant frequency obtained by the model is 47.65 kHz (the error is 10.81%).

This model allows only the thickness resonant modes to be predicted and, therefore, does not consider the inevitable radial resonant modes. One-dimensional models are generally not suitable for determining resonant frequencies of thickness oscillations that are close to resonant frequencies of radial oscillations. In the case shown, when the model does not predict the third and fourth modes, the calculated resonant frequencies for the first two modes are always higher than the measured ones. If a model that considers both the third and fourth modes were used, the first two modes would be moved to lower frequencies.

IV. Conclusion

In this study, an equivalent circuit was developed for accurate analysis of the acoustic characteristics of an ultrasonic transducer over a wide frequency range.

Eqs. (1-4) confirm that the frequency characteristics of transducers in one-dimensional theory depend on the material characteristics of the components of composite transducers and their geometric dimensions.

In practice, one-dimensional modeling is most often used due to the great flexibility and efficient implementation of the model. The flexibility and efficiency of one-dimensional models come to the fore in the analysis of transducers operation, which includes a large number of parameters.

The proposed one-dimensional model of composite transducer does not include mechanical and electrical losses in the material. However, losses can be analyzed if the piezoelectric constants and the elastic constants of the metal parts of the converter are represented in complex numbers, where their imaginary parts represent losses.

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