On Pulse Shaping for Generalized Faster than Nyquist Signaling with and without Equalization

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***Abstract*—This paper focuses on analyses of generalized Faster than Nyquist (FTN) signaling in the presence of additive white Gaussian noise. A new method for designing pulse shaping filters, that maximize information rate and simultaneously obey constraints related to energy distribution of the pulse autocorrelation function, is proposed. The obtained pulses are coupled with the minimum mean square error (MMSE) equalizer used at the receiving side. In addition, potential for their use without any equalization scheme is also analyzed. Significance of the proposed approach is verified by comparing designed pulses with state-of-the-art FTN schemes, that employ raised cosine pulses, in terms of bit error rate and achievable information rate. We identify cases when the proposed scheme provides the same achievable information rate as the standard FTN system with more than 1.5 dB lower signal-to-noise power (SNR) ratio, without the equalization, and 0.4 dB lower SNR ratio, if MMSE equalization is employed.**

***Index Terms*—Faster than Nyquist signaling, MMSE equalization, pulse shaping**

I. Introduction

In a classical communication system information is transmitted by using orthogonal pulses (with flat frequency spectrum) and ideally no inter-symbol interference is introduced by the transmitter, which is refereed to as the Nyquist signaling approach. Intense research over the past years in the area of channel coding and modulation lead to development of Nyquist transmission systems that operate close to the ultimate Shannon spectral efficiency bounds, and obviously, additional improvement must be followed by the change of Nyquist's transmission paradigm. With the invention of new services, mostly associated with the fifth generation standard for broadband cellular networks (5GNR), the problem of designing spectral efficient transmission system comes again under the spotlight. A technique that is capable to provide a quantum leap in design of spectrally efficient systems is faster then Nyquist (FTN) signalling.

In FTN signaling systems use of orthogonal pulses is abandoned and inter-symbol inference is intentionally introduced. It follows that as the adjacent received symbols are correlated, conventionally used symbol-by-symbol detection becomes inappropriate, and information needs to be extracted by some equalization technique. The foundations of FTN were laid down by Mazo in 1975 [1], who noticed that communicating with symbol rates higher than the Nyquist rate, can provide spectrally efficient transmission, without degradation in Euclidean distances between transmitted symbols. Although the aforementioned insight was revolutionary, it was not fully explored for more than 30 years. Namely, in 2007 Rusek and Anderson [2] created the information-theoretic framework for the analysis of FTN systems and proved that capacity of conventionally used Nyquist systems (for example with raised cosine (RC) pules) can be surpassed with FTN signaling. Furthermore, the same authors showed in [3] that, by increasing the symbol transmission rate, constrained capacity saturates to a fixed value – in other words arbitrary spectral efficiency can be achieved. Their work was refined recently by Ishihara and Sugiura in [4], where it was shown that conventionally used RC pulses employed in precoded FTN systems approach Shannon capacity of the ideal rectangle pulse. For excellent overview on FTN concepts and technologies we direct readers to [5].

In order to keep the advantages of FTN over Nyquist signaling in practical systems, the equalization and channel coding need to be adjusted. The optimal FTN receiver is organized in a form of turbo equalization loop [6], where equalization is performed by employing maximum a posteriori probability (MAP) detector. Complexity of MAP detection grows exponentially with increase of symbol rate, making it infeasible for practical use. On the opposite side, using low complex equalization, for example MMSE (Minimum Mean Square Error), may be insufficient to perform significantly beyond conventional Nyquist systems. This lead to *generalized FTN signaling* approach [7], in which conventional pulses, like RC, are replaced with pulses that are adjusted to a given equalization scheme. Another benefit of custom pulse design is ability to adopt to a given practical requirements, for example peak-to-average power ratio (PAPR) or adjacent channel leakage power (ACLP), which can vary from one communication standard to another.

Over the years, different pulse designs were proposed that can be incorporated into generalized FTN concept. The most prominent approaches include pulses designed: i) to minimize Euclidean distance between different realizations of two random transmitted sequences [8] and bit error rate of uncoded transmission [9], ii) to maximize information rate [2], [10], [11], or iii) to obey predefined frequency domain constraints (and simultaneously limit PAPR) [12]. For example, Rusek and Anderson in [2] proposed an optimization procedure in which pulses with predefined number of taps maximize an upper information rate bound. The procedure was further expanded by Brkic *et al.* in [11] to enable arbitrary

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pulse energy distribution in time domain, which can be used to build generalized FTN systems with limited trellis-based equalizer complexity.

In this paper we further extend the optimization procedure presented in [2], [11] in order to make it applicable to FTN systems with MMSE equalizers, or even to FTN transmission systems which do not employ any equalization scheme. Namely, we define additional time domain constraints related to energy of the pulse autocorrelation function. We verify that designed pulses outperform state-of-the-art FTN systems (with RC pulses), for the same symbol rate and ACLP, in terms of bit error rate as well as achievable information rate in additive white Gaussian noise (AWGN) channel.

The rest of the paper is organized as follows. In Section II we briefly describe system model and MMSE equalization scheme. Section III is dedicated to the optimization procedure, while numerical results are given in Section IV. Finally, concluding remarks can be found in Section V.

II. System Model

Consider an output of the baseband equivalent of the transmitter

$$s(t) = \sum_{k=-\infty}^{\infty} a_k h(t - kT), \tag{1}$$

where $a_k \in \{\pm 1\}$ corresponds to the k-th transmitted symbol, T denotes symbol duration and $h(t)$ is the pulse shaping filter. We assume that $h(t)$ is not orthogonal with respect to the transmitting sample rate, i.e, it intentionally introduces ISI and that $h(t)$ can be represented as weighted sum of wider band pulses

$$h(t) = \sum_{l=0}^{L-1} b_l \psi(t - lT), \tag{2}$$

where $\psi(t)$ is a pulse orthogonal to the sampling rate $1/T$, while $\mathbf{b} = (b_0 \ldots, b_{L-1})$ corresponds to a vector of the sampled coefficients, i.e, $b_l = h(lT)$. Note that we restrict the effect of ISI to L consecutive symbols and that energy of the impulse response is considered to be unitary.

The waveform $s(t)$ is transmitted through additive white Gaussian (AWGN) channel, described with energy per symbol to noise power spectral density (E_s/N_0) metric.

At the receiver side in this paper we consider two observation models: i) orthogonal basis model (OBM) [13] and ii) Ungerboeck model [14] without the equalization. According to the OBM, receiving filter is matched to $\psi(t)$ (not $h(t)$), which means that noise at the receiver input is white. In our model we assume that $\psi(t)$ is square root raised cosine pulse with roll-off 0.1. To verify the performance of $h(t)$ on the OBM, we employ MMSE equalizer. Motivated by the recent findings, reported in [15], that FTN signaling can perform satisfactory even without an equalizer implemented in the receiver side, we here consider such detection scheme, for the case of the Ungerboeck observation model. In the Ungerboeck model receiving filter is matched to $h(t)$, which maximizes E_s/N_0 metric; however, received noise sequence becomes correlated.

We next briefly explain MMSE equalization used in the OBM (for more details one could see [16]). Let $\mathbf{r}_n = (r_{n-N_2}, r_{n-N_2+1}, \ldots, r_{n+N_1})^T$ denote a sequence at input of the MMSE equalizer used to estimate transmitted symbol a_n, where T is transposition sign. The parameters N_1 and N_2 specify the length of the noncausal and the causal part of the MMSE filter. Then, finite impulse response (FIR) MMSE filter coefficients $\mathbf{c}_n$ can be obtained as follows

$$\mathbf{c}_n = \mathrm{Cov}(\mathbf{r}_n, \mathbf{r}_n)^{-1} \times \mathrm{Cov}(\mathbf{r}, a_n), \tag{3}$$

where the covariance operator is given by $\mathrm{Cov}(\mathbf{x}, \mathbf{y}) = E(\mathbf{x}\mathbf{y}^H) - E(\mathbf{x})E(\mathbf{y}^H)$, where $E(\cdot)$ denotes mathematical expectation, and H is Hermitian operator. The symbol estimate $\hat{a}_n$ is obtained by $\hat{a}_n = \mathbf{c}_n^H \mathbf{r}_n$. It should be noted that values N_1 and N_2 are dependent on $h(t)$.

III. Optimization of pulse shaping filter

In this section we state the optimization problem for finding pulse shaping filter coefficients $\mathbf{b}$ that maximize the achievable information rate (AIR) and are also adjusted to the equalizing scheme. Fundamentals of the AIR-based optimization can be found in [17], and we first briefly explain key concept of the optimization, and then highlight modifications introduced in order to adjust the optimization procedure to the system model, given in Section II.

The procedure from [17] allows pulse design for arbitrary ACLP value and the filter length L. Let $H(f)$ denotes Fourier transform of $h(t)$. Then we can define, a complement of ACLP, the concentration of pulse energy in W Hz as follows

$$\beta = \frac{\int_{-W}^{W} |H(f)|^2 df}{\int_{-\infty}^{\infty} |H(f)|^2 df}. \tag{4}$$

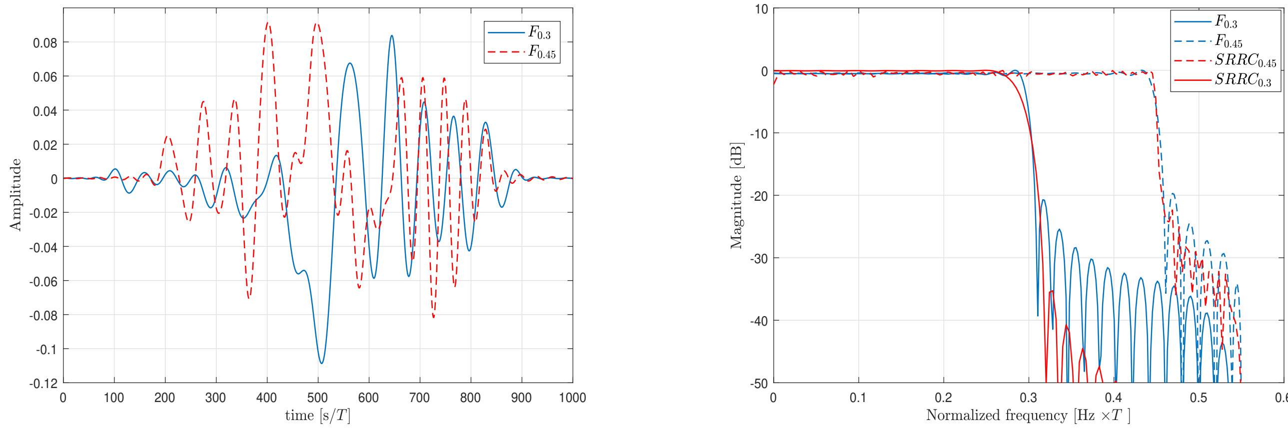
Without loss of generality, we can consider normalized bandwidth defined as $w = W \times T$, and notice that $w = 0.5$ corresponds to Nyquist signaling while when $w < 0.5$ we are communicating in FTN signaling fashion. The amount of ISI introduced at transmitter side is inversely proportional to w. Alternatively, the energy concentration β can expressed in a more suitable form, as a function of pulse discrete autocorrelation function $\mathbf{g} = (g_{-L+1}, \ldots, g_{L-1})$, as follows [11]

$$\beta(\mathbf{g}, w) = \sum_{l=-L}^{L} 2 g_l w \times \mathrm{sinc}(2\pi w), \tag{5}$$

where $\mathrm{sinc}(x) = \sin(x)/x$, and

$$g_l = \int_{-\infty}^{\infty} h(t) h^\star(t - lT) dt. \tag{6}$$

Direct maximization of AIR, for a given discrete modulation set is infeasible as, to the best of our knowledge, a closed form expression does not exist. Instead, it is common to optimize



(a)

(b)

Fig. 1: Impulse (a) and (b) frequency responses of designed filters for $w = 0.3$ and $w = 0.45$ ($L = 50$, $\beta = 0.999$).

an information rate upper bound $C(\mathbf{b})$, derived assuming Gaussian sample distribution [17]

$$C(\mathbf{b}) = \int_0^{1/2} \log_2 \left[1 + \frac{2E_s|B(f)|^2}{N_0 T} \right] df, \tag{7}$$

where $B(f) = \sum_{k=0}^{L-1} b_k e^{i2\pi kf}$ represents Fourier transform of the vector $\mathbf{b}$.

It was reported in [11] that in large number of cases $C(\mathbf{b})$ monotonically increases with actual AIR obtained through computationally hungry Monte Carlo simulation, which means that optimizing $C(\mathbf{b})$ is meaningful.

By studying typical behavior of pulses optimized by (7), for a fixed β and w constraints we noticed the following: i) relatively small number of filter taps is sufficient to obey strict b constraint (for example $\beta = 0.999$) and ii) significant portions of energy of the pulse autocorrelation are spread across a large number of taps, i.e., the account of introduced ISI is large. Obviously, such pulses cannot be used in systems that do not use equalizers, and are inadequate when equalization is performed with the MMSE equalizer, given its modest ability to suppress ISI.

To resolve the aforementioned issue, we proposed that additional constraint is added into optimization setup that will force the optimization procedure to cluster the majority of the autocorrelation energy to main tap g_0 and potentially $2M$ adjacent taps $(g_{-M}, , g_{-1}, g_1, \dots g_M)$. Namely, we define a set of thresholds f $0 < f_i < 1$, $0 \leq i \leq M$, forcing the relative energy of central autocorrelation taps to be above the thresholds. However, given frequency-time duality principle, clustering autocorelation energy makes it harder to satisfy frequency domain constraint β. To overcome the problem, the filter lengths L must be increased.

For predefined normalized bandwidth w, with energy concentration β_0 we formally express the optimization problem as follows

$$\begin{aligned} \mathbf{b}_{opt} &= \underset{\mathbf{b}}{\operatorname{argmax}}\ C(\mathbf{b}) \\ s.t.\ \ &\beta(\mathbf{g}, w) = \beta_0, \\ &\frac{g_i^2}{\sum_{\ell=-L+1}^{L-1} g_\ell^2} \geq f,\ 0 \leq i \leq M. \end{aligned} \tag{8}$$

The above optimization problem can be solved similarly as the related problems described in [17] and [11], by sequential quadratic programming (SQP) method.

It should be noted that in OBM system described in Section II, we do not match the receiving filter to transmitting $h(t)$, which means that amount of ISI collected by the receiver is not directly expressed by autoccorelation of $h(t)$. However, we observed strong dependency between ability of MMSE equalizer to suppress ISI and the energy concentration of the pulse autocorrelation function.

To illustrate our optimization procedure, we designed two pulses with $\beta = 0.999$, $L = 50$ and $w = 0.3$ and $w = 0.45$, respectively, denoted by $F_{0.3}$ and $F_{0.45}$, design to obey $f_0 = 0.58$ and $f_0 = 0.85$, respectively. Their impulse and frequency responses are depicted in Fig. 1. For compression we also give frequency responses of square root raised cosine (SRRC) pulses with roll-offs equal to 0.1, designed to meet the same requirements as optimized pulses in terms of length and energy concentrations in frequency domain (denoted by $SRRC_{0.3}$ and $SRRC_{0.45}$).

IV. Numerical results

In this section we provide the performance of designed pulses $F_{0.3}$ and $F_{0.45}$, in terms of bit error rate and achievable information rates, obtained by Monte Carlo simulation (Figs. 2 and 3). We examine pulses behaviour on OBM and Ungerboeck system models, introduced in Section II. Obtained results are compared to SRRC pulses ($SRRC_{0.3}$ and $SRRC_{0.45}$). Given the fact that we only consider binary

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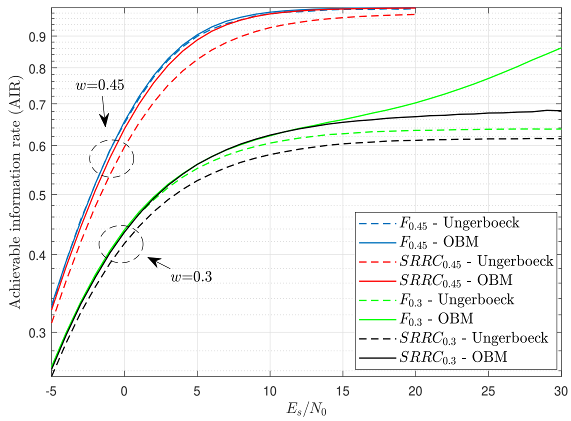

Fig. 2: Achievable information rates of designed pulses.

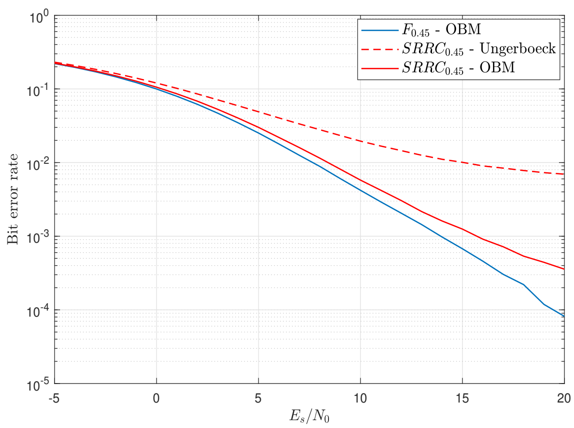

Fig. 3: Bit error rate achievable by $F_{0.45}$ filter.

transmission ($a_k \in \{\pm 1\}$), achievable information rates are calculated numerically as follows

$$AIR = \frac{1}{2} \sum_{a_k=-1,1} \int_{-\infty}^{+\infty} p(z|a_k) \times \log_2 \frac{2p(z|a_k)}{p(z|1) + p(z|-1)} dz, \tag{9}$$

where the conditional probability density function of the log likelihood ratio z of the symbols that corresponds to transmitted a_k, $p(z|a_k)$, is approximated by a histogram.

In Fig. 2 we show that designed pulses outperform SRRC counterparts in terms of AIR on the both system models. For example, $F_{0.45}$ achieves AIR, equal to 0.8 bits/symbols, with 0.4 dB less E_s/N_0 value compared to $SRRC_{0.45}$ on the OBM, while the differences on the Ungerboeck model is approximately 1.8 dB. One can also notice that $SRRC_{0.3}$ cannot achieve information rates above 0.68 bits/symbol in the OBM, while if $F_{0.3}$ is used, higher spectral efficiencies are possible. If we consider uncoded transmission we can, notice that $F_{0.45}$ achieves bit error rate of 10^{-3} with 1.7 dB less E_s/N_0 compared to $SRRC_{0.45}$ on the OBM (Fig. 3).

V. Conclusion

This paper provides novel pulse shaping filters applicable to generalized FTN signaling systems with MMSE equalization and also to systems with no equalizer employed. We show that proposed pulses outperform SRRC pulses of the same structural properties. Our future work will be oriented into examining finite length coded transmission systems that employ proposed pulses.

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