Performance simulation for LCR of MIMO Multi-branch SC Diversity System in α-μ fading and α-μ interference channel

Dejan Milić, Suad Suljović, Dejan Rančić, Nenad Petrović, Nenad Milošević

Abstract - **In order to improve the overall performance of the network in 5G telecommunication networks, Multiple Input and Multi Output Technology (MIMO) is applied. In this paper, the mean number of Level Crossing Rate (LCR) of MIMO systems with L-branch selection combining (SC) receiver is analyzed. During signal transmission, its distortion occurs due to low α-μ fading and α-μ co-channel interference (CCI) effects. Additionally, we applied an accelerated graphics processing unit (GPU) simulation to plan a QoS-efficient 5G mobile network in a smart city. This approach in combination with linear optimization and deep learning significantly optimizes the LCR calculation speed for the observed communication system type, while providing efficient planning - reducing costs, but maximizing performance.**

***Index Terms*—SC combining, α-μ fading, α-μ interference, LCR, QoS, GPU, linear optimization.**

I. Introduction

Due to the unpredictable and dynamic nature of the wireless channel environment, the channel becomes one disruptive element in the transmission chain as it changes the broadcast signal. Therefore, the channel manages the performance of wireless communication systems [1].

Multiple Input Multiple Output (MIMO) is an efficient antenna technology for wireless communication systems where multiple antennas are used on the transmitter and receiver [2]. Antennas at each end of the system are combined to minimize errors during signal transmission, increase data rates and improve channel capacity. This technique allows signals to be transmitted on many different paths at the same time. This mode of signal transmission provides the ability for signals to reach the receiver without fading and co-channel interference. This technique increases the signal-to-noise ratio (SNR) and transmission quality, which creates more stable connections [3].

Fading is a variation of attenuation, ie. signal amplitude fluctuations. During transmission, the signal faces various obstacles such as buildings, trees, etc. present in the signal-causing environment is subject to reflection, diffraction, scattering and shading. This presence of multiple reflectors in the environment of the channel between the transmitting and receiving ends creates more paths for signal passage [4].

The most commonly used signal processing techniques in diversity systems are maximum ratio combination (MRC), equal gain combination (EGC), and selection combination (SC). MRC provides the best improvement in system performance, followed by an equal combination of reinforcements, but it is also the most complicated technique. In order to reduce the complexity of the receiver, this paper considers a simpler combination scheme related to combination selection (SC). The output of the SC receiver is the branch with the highest signal-to-noise ratio. SC has been extended to the case where signals on more than one receiving antenna are combined with the highest current SNR, this scheme is called hybrid maximum ratio selection/combination (HS/MRC). Selection Combination (SC) techniques have been applied in the design of MIMO systems to reduce system complexity and costs. Fifth generation (5G) telecommunications offers a 10-fold increase in spectral efficiency and a 1000-fold increase in system capacity compared to 4G technology [5]. In MIMO systems, there are tens to hundreds of antennas in the receiver and transmitter. Massive MIMO techniques use familiar channel features to deliver superior performance in wireless communications. Channels are modeled to include channel variations in frequency and time.

Describing and modeling channels with fading is of particular importance in mobile communications both for the design of the transceiver system and for performance analysis. During the long period of development of wireless communications, a large number of different models of channels with fading were constructed to describe the statistics of the envelope and the phase of the channel where the signal propagates in several paths, [6]. Examples of such models are Rayleigh's, Rice's, Nakagami-q, Nakagami's, Weibull's, Beckmann's, $\alpha - \mu$ etc. The aim of this paper is to study the statistical properties of first and second order

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envelopes and phases in these models, with with special reference to the α − μ model.

In mobile communications, the received signal varies in a wide range of values. Therefore, for the design of digital and analog systems, it is necessary to know the statistical characteristics of the signal. There are first and second order statistical characteristics. It is especially necessary to know the statistical characteristics of the second order, such as the average number of axial sections of the LCR and the average duration of AFD fading. These quantities provide additional information that, when combined with other statistics, allows designers to create rational system solutions [7].

Level Crossing Rate (LCR) and Average Duration of Fades (ADF) are important second-order statistical characteristics for describing fading channels. These quantities are useful in the design of mobile radio communication systems and for the analysis of their performance. In digital telecommunications, a sharp drop in the envelope value of the received signal directly leads to a sharp increase in the probability of error [8]. Second-order statistics calculation techniques are particularly applicable to diversity systems that have been shown to be very useful in reducing the impact of fading.

In this paper, we consider a 5G communication system that works over k-ading fading channels and k-μ co-channel interference. One of the most basic statistics of a wireless communication system operating in a fading environment is the mean number of axial cross-sections (LCR). The quality of service (CoS) in wireless communications depends significantly on the LCR as it allows the estimation of the minimum distance between two base stations. We first compute the cumulative distribution function (CDF) for the signal in the described environment, and then we derive a closed-form expression for the LCR. The results of this analysis can be used to design an optimal receiver for a 5G mobile network in smart cities in conditions of small α-μ fading and α -m interference.

II. LCR OF SIGNAL TO INTERFERENCE RATIO AT THE OUTPUT OF THE L-BRANCH SC RECEIVER

In this section, the statistics of the second order 5G wireless communication system with SC receiver with L branches are considered. The received desired signal experiences α-μ fading, while the interference on the co-channel is subjected to α-μ fading. The model of the receiver is shown in Fig. 1.

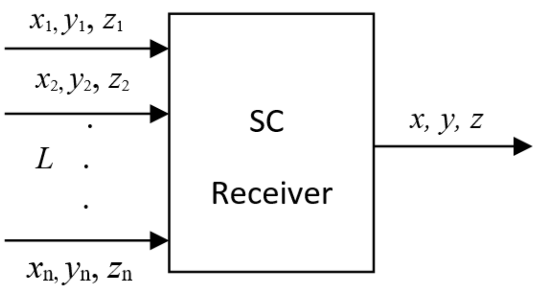

Fig. 1. The model of the SC receiver with *L* branches

Signals $x_1, x_2, ..., x_n$ come to the inputs of the SC combiner. The SC receiver with L input branches *(L = 2, 3, ... n)* selects the signal from the antenna with the highest SNR. Also, interference envelopes in the co-channel interference appear as $y_1, y_2, ..., y_n$ at each of the L inputs of the SC receiver, while the corresponding output signal is y. The PDF signal envelope on input at SC receiver have the α-μ distribution [9]:

$$p_{x_i}\left(x_i\right) = \frac{\alpha\mu^{\mu} x_i^{\alpha\mu-1}}{\Omega_i^{\mu}\Gamma\left(\mu\right)} e^{-\mu\frac{x_i^{\alpha}}{\Omega_i}} \tag{1}$$

where α is a positive parameter, Ω_i is the mean value of the signal power, μ is the number of clusters and Γ(·) is a Gamma function. In a described channel, the co-channel interference signal follows α-μ distribution:

$$p_{y_i}\left(y_i\right) = \frac{\alpha\mu^{\mu} y_i^{\alpha\mu-1}}{s_i^{\mu}\Gamma\left(\mu\right)} e^{-\mu\frac{y_i^{\alpha}}{s_i}} \tag{2}$$

where *s* is average power of *y*, $y_i \geq 0$. The ratio of the desired signal envelope and interference on the *i*-th input branch of the SC receiver with the L branch can be written as:

$$z_i = \frac{x_i}{y_i}, x_i = z_i y_i \tag{3}$$

SNR for *i = 2.3,… n* at the output of the SC receiver is:

$$z = \max\left(z_1, z_2 ... z_i\right) \tag{4}$$

The probability density function (PDF) of the signal z_i is given by [10]:

$$p_{z_i}\left(z_i\right) = \int_0^{\infty} \mathrm{d}y_i y_i p_{x_i}\left(z_i y_i\right) p_{y_i}\left(y_i\right) = \frac{\alpha z_i^{\alpha\mu-1}\left(\Omega_i s_i\right)^{\mu}\Gamma\left(2\mu\right)}{\Gamma^2\left(\mu\right)\left(\Omega_i + s_i z_i^{\alpha}\right)^{2\mu}} \tag{5}$$

Cumulative distribution function (CDF) of z_i is [10]:

$$F_{z_i}\left(z_i\right) = \int_0^{z_i} \mathrm{d}t p_{z_i}\left(t\right) = \frac{\Gamma\left(2\mu\right)}{\Gamma^2\left(\mu\right)} B_{\frac{s_i z_i^{\alpha}}{\Omega_i + s_i z_i^{\alpha}}}\left(\mu, \mu\right) \tag{6}$$

where $B_z(a, b)$ is the incomplete Beta function, [11; 8.39]. SNR for *i* = 2.3, … *n* will be at the output of SC combination:

$$z = \max\left(z_1, z_2 ... z_i\right) \tag{7}$$

First derivative of z_{ij} can be written:

$$\dot{z}_{ij} = \frac{1}{y_{ij}}\dot{x}_{ij} - \frac{x_{ij}}{y_{ij}^2}\dot{y}_{ij} \tag{8}$$

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The derivative of the random process α-k-μ is a Gaussian random process, and a linear combination of Gaussian processes is also a Gaussian random process. Therefore, the conditional Gaussian distribution $\dot{z}$ with zero mean and variance applies:

$$\sigma^2_{\dot{z}_{ij}} = \frac{1}{y_{ij}^2}\sigma^2_{\dot{x}_{ij}} + \frac{x_{ij}^2}{y_{ij}^4}\sigma^2_{\dot{y}_{ij}} \tag{9}$$

where respective variances relating to signal and interference are [12]:

$$\sigma^2_{\dot{x}_{ij}} = \left(\frac{2\pi f_m}{\alpha}\right)^2 \frac{\Omega_i x_{ij}{}^{2-\alpha}}{\mu}, \sigma^2_{\dot{y}_{ij}} = \left(\frac{2\pi f_m}{\alpha}\right)^2 \frac{s_i y_{ij}{}^{2-\alpha}}{\mu} \tag{10}$$

where f_m denotes the Doppler frequency. After replacing expression (12) with (11), the variance $\dot{z}_{ij}$ becomes:

$$\sigma^2_{\dot{z}_i} = \frac{1}{y_i^2}\sigma^2_{\dot{x}_i} + \frac{x_i^2}{y_i^4}\sigma^2_{\dot{y}_i} = \frac{1}{\mu y_i^{\alpha} z_i^{\alpha-2}}\left(\frac{2\pi f_m}{\alpha}\right)^2 \left(\Omega_i + z_i^{\alpha} s_i\right) \tag{11}$$

In equation (13), fm denotes the Doppler frequency. Conditional probability density functions (CPDF) of $\dot{z}_i$ and z_i are [13]:

$$p_{\dot{z}_i}\left(\dot{z}_i \middle| z_i y\right) = \frac{1}{\sqrt{2\pi}\sigma_{\dot{z}_i}} e^{-\frac{\dot{z}_i^2}{2\sigma^2_{\dot{z}_i}}},$$

$$p_{z_i}\left(z_i \middle| y_i\right) = \left|\frac{\mathrm{d}x_i}{\mathrm{d}z_i}\right| p_{x_i}\left(z_i y_i\right) = y_i p_{x_i}\left(z_i y_i\right) \tag{12}$$

Conditional joint probability density function (CJPDF) of z_i, $\dot{z}_i$ and y_i is [13]:

$$p_{\dot{z}_i z_i y_i}\left(\dot{z}_i z_i y_i\right) = p_{\dot{z}_i}\left(\dot{z}_i \middle| z_i y_i\right) p_{z_i}\left(z_i \middle| y_i\right) p_{y_i}\left(y_i\right) =$$

$$= p_{\dot{z}_i}\left(\dot{z}_i \middle| z_i y_i\right) p_{y_i}\left(y_i\right) y_i p_{x_i}\left(z_i y_i\right) \tag{13}$$

At the output of the SC receiver, the LCR signal is calculated as the mean value of the first derivation of the signal at the output of the SC receiver. The joint probability density function (JPDF) of z_i and $\dot{z}_i$ [13]:

$$p_{\dot{z}_i z_i}\left(\dot{z}_i z_i\right) = \int_0^{\infty} \mathrm{d}y_i\, p_{\dot{z}_i z_i y_i}\left(\dot{z}_i z_i y_i\right) \tag{14}$$

By integrating the first derivative, averaging is obtained. The rate of transition of the LCR level of the random process zi is [10]:

$$N_{z_i}\left(z_i\right) = \int_0^{\infty} \mathrm{d}\dot{z}_i \dot{z}_i p_{\dot{z}_i z_i}\left(\dot{z}_i z_i\right) =$$

$$= \int_0^{\infty} \mathrm{d}\dot{z}_i \dot{z}_i \int_0^{\infty} \mathrm{d}y_i p_{\dot{z}_i}\left(\dot{z}_i \middle| z_i y_i\right) p_{y_i}\left(y_i\right) y_i p_{x_i}\left(z_i y_i\right) =$$

$$= \frac{\sqrt{2\pi}\, f_m z_i^{(2\alpha\mu-\alpha)/2}\left(s_i\Omega_i\right)^{(2\mu-1)/2}\Gamma\left(\left(4\mu-1\right)/2\right)}{\Gamma^2\left(\mu\right)\left(s_i z_i^{\alpha} + \Omega_i\right)^{2\mu-1}} \tag{15}$$

The LCR envelope of the signal-to-interference ratio at the output of the mD SC with n inputs is [14]:

$$N_{x_i|\Omega_i s_i}\left(z_i\right) = L\left(F_{z_{ij}}\left(z_{ij}\right)\right)^{L-1} N_{z_{ij}}\left(z_{ij}\right) = \frac{L\sqrt{2\pi}\, f_m z_i^{\frac{2\alpha\mu-\alpha}{2}}}{\Gamma^{2L}\left(\mu\right)}$$

$$\frac{\left(s_i\Omega_i\right)^{\frac{2\mu-1}{2}}\Gamma^{L-1}\left(2\mu\right)\Gamma\left(\frac{4\mu-1}{2}\right)}{\left(\Omega_i + s_i z_i{}^{\alpha}\right)^{2\mu-1}}\left(B_{\frac{s_i z_i{}^{\alpha}}{\Omega_i + s_i z_i{}^{\alpha}}}\left(\mu,\mu\right)\right)^{L-1} \tag{16}$$

Fig. 2 shows a graphical analysis of the LCR at the SC receiver outputs given, where, $\Omega_1=\Omega_2=\ldots\Omega_n$, and $s_1=s_2=\ldots s_n$; It is assumed that the correlation between the input branches in the SC receiver is minimal.

III. NUMERICAL AND GRAPHICAL RESULTS

Based on expression (16) in Fig. 2 the LCR signal-to-noise ratio is shown in relation to the transition threshold at the SC receiver output from the L branches.

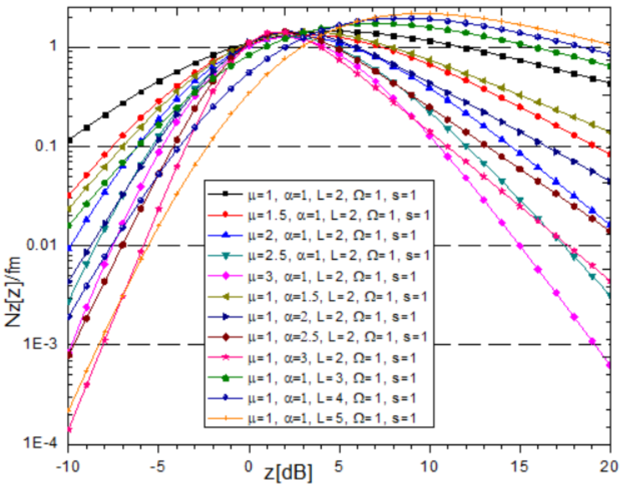

Fig. 2. LCR of system, for different values of parameters μ, α and L.

Fig. 2 shows that for higher values of the signal-to-noise ratio, the increase in the value of the μ parameter LCR decreases. When the parameter α increases then comes to narrowing the LRC function. As the number of L branches at the combinator input increases, the LCR increases by positive values of z [dB], and the system has better performance.

IV. Planning and Simulation Environment

In this section, we present how the previously derived LCR expression can be leveraged within model-driven network planning environment making use of GPU hardware and multi-objective optimization built upon our previous works [15, 16, 17]. Workflow of the underlying software environment exploiting the synergy of deep leaning and multi-objective optimization is depicted in Fig. 3.

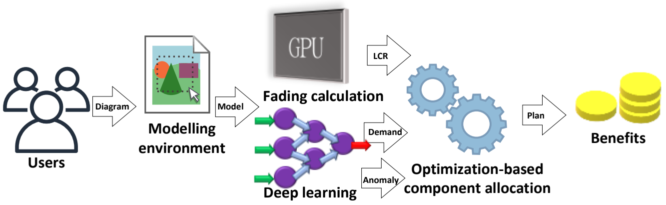

Fig. 3. LCR of system, for different values of parameters μ, α and L.

Within the first step, users construct network diagram inside Eclipse-based environment, according to the structure of underlying network planning metamodel [15, 16]. It considers the following aspects relevant to planning of mobile network inside smart city: operator's base stations, terrain configuration with obstacles, properties of communication channel, adaptive behavior in case of base station anomalies, consumers of telco services (including people equipped with smartphones, autonomous vehicles and smart city infrastructure).

Once modelling is done, the created model is parsed and processed using Ecore [18]. Furthermore, the corresponding parameters are used as input of network-related data processing steps: GPU-enabled LCR determination and deep learning-based predictions. For purpose of fast LCR calculation, we wrote NVIDIA CUDA [19] kernels in C programming language, as described in [17]. In our experiments, it was up to 65.5 times faster than CPU-only program in Mathematica.

On the other side, deep learning module was implemented relying on PyTorch [20] in Python programming language and deployed as Flask service. It covers predictions related to the following factors: number of service users and base station anomalies. For service user count prediction number day, daily temperature and special occasion flag (such as holiday) are considered. On the other side, for base station anomalies ratio prediction the following factors are involved: total downloaded and uploaded data, Quality of Service value, energy consumption and number of users (previously predicted). The layouts of datasets together with achieved prediction performance (Mean Relative Error) are shown in Table I. The problems are treated as regression (real-valued outcome determination), For this purpose, we designed multi-layer perceptron (MLP) with 3 hidden layers, 30 nodes each, making use of Adam optimizer with learning rate 0.01 for training.

TABLE I
Overview of Prediction Models

Model	Input	Approach	Relative error
User number	Day Special Temperature	Regression 3 hidden 30 nodes Adam α=0.01	16%
Anomaly Ratio	Users Download Upload QoS Energy		13%

Finally, in the last step, we make use of our optimization-based model-driven framework for component allocation relying on Pymoo [21], with respect to the model which will be described. The goal of optimization procedure is finding network plan that consists of optimal base station ($bs \in B$) placement for desired smart city locations ($p \in P$), maintaining best possible QoS while keeping lowest costs at the same time. The objective function has following form:

$$minimize \sum_{bs \in BS, l \in L} plan[l,bs](LCR[l,bs] + cost[l,bs] + anomaly[l,bs]) \quad (17)$$

As it can be seen, the sum of LCR (positive impact on performance for lower values), deployment costs and base station anomaly rato is minimized. Here, *plan[l, bs]* denotes decision variable which is 1 in case if base station bs is going to be placed on location l, otherwise 0.

Additionally, we apply a constrint for each location *l* that capacity of base station *bs* (refered to as *maxu[l,bs]*) should be enough to service the predicted number of users (*demandp[l]*) for given location *l*.

$$\sum_{bs \in BS} plan[l,bs] maxu[l,bs] \geq demandp[l], l \in L \quad (18)$$

V. Conclusion

In this paper, we have analyzed the MIMO system with a multi-branch SC receiver with L branches when coming to system inputs α - μ fading and CCI with the same distribution. We performed LCR for this model of receiver in the presence of the above-mentioned fading and interference. System performance is more favorable in case when system has lower μ and α parameter, for lower SIR. Analyzing the system, we noticed that the performers are better when the system has a larger number of input branches, because the combiner has the ability to select the branch with the best SIR, leads to better system performance. Finally, for the above fading and CCI system performance is more favorable for lower values of average LCR.

Use of the obtained expression for LCR at the output of a multifaceted SC combinator, we have suggested software environment for simulation. There are several benefits: 1)

GPU reduces time for LCR calculation 2) costs are minimized 3) performance is maximized.

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