Reliability of Earth-Space Links under Deep Fades with Interleaved Reed-Solomon Codes

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***Abstract*—This paper contains a reliability analysis of Earth-space links subjected to deep fades, modeled as burst erasure channels. We numerically calculate lower bounds on transmission propagation latency caused by employment of packet erasure codes, when fade duration is represented by random variable with Weibull distribution. Furthermore, we propose coding scheme that involves interleaved short Reed-Solomon (RS) codes to mitigate information loss, caused by long fading events. In order to quickly and accurately evaluate residual packet loss rates of interleaved RS codes, we construct a novel simulator, called segment-based simulator, which is able to predict code performance several orders of magnitude faster than plain Monte Carlo simulation. Finally, we show that for variety of channel parameters and code rates, very short RS codes (even with length 15) can provide near optimal propagation latencies.**

***Keywords*—Fading channels, Packet erasure codes, Reed-Solomon codes, Earth-space Links, Singleton bound**

I. Introduction

Satellite communications are experiencing a renaissance over the past years, as their potential in providing broadband transmission is fully recognized and explored. Possibility to transfer information between arbitrary points on the Earth surface via satellites makes this type of communications attractive for emerging services, which require high accessibility, like connected vehicles. Furthermore, inclusion of satellite links into 5G ecosystem is studied in newly published 3GPP standards [1], in an attempt to create powerful hybrid satellite-terrestrial (HST) communication systems [2]. In order to build HST systems different technological challenges need to be resolved, which are partially consequences of reduced accessibility and reliability of Earth-space channels.

Despite widely assumed line-of-site visibility between ground terminals and satellites, Earth-space communications link could be blocked for a variety of reasons. In mobile satellite communications used in urban areas, line-of-site assumption may not hold, while different ground obstacles could, from time to time, disable the communication link, leading to loss of transmitted information. Furthermore, LEO (Low Earth Orbit) satellites are moving along their orbits and each LEO satellite is only shortly visible from a stationary point at Earth surface. To enable uninterrupted transmission, ground terminal periodically executes handover operations and establishes connection with different LEO satellite. Handover operation can not be instantaneously performed, which means that potentially communication channel is for a period of time blocked. Lastly, meteorological conditions, for example heavy rain, could attenuate signal such that its power cannot reach a receiving threshold.

All the above phenomenons can be described via hard-blockage model [3], where on-state (reliable transmission) and off-state (blocked channel with no transmission) are alternatively changed, while state durations are represented as uncorrelated random events. Probability density functions of state durations are chosen to fit empirical data collected over the years, mostly for Ka frequency band [4], [5].

Reliability of Earth-space links could only be improved by forward error correction, as propagation delay prohibits use of retransmission techniques. Given the fact that hard-blockage model corresponds to block erasure channel, using sufficiently long maximum distance separable (MDS) codes, for example Reed-Solomon codes, will be optimal. However, for high throughput communications, during off-states the large amount of symbols are erased, and consequently length of a required MDS code must be high (usually measured in tens of Mb or higher). Decoding complexity of MDS codes increases quadratically with code length, which makes them impractical in Earth-space channels. Instead, suboptimal solutions are of interest, like interleaved RS codes [6] and Raptor codes [7]–[9], whose decoding complexity increases only linearly with code length, or LT (Luby Transform) codes [10], with the worst case $O(n \log n)$ complexity.

Possibility of increasing reliability of Earth-space links with LT codes was analyzed in [11], [12]. The authors in [12] showed that UDP transport protocol equipped with LT codes outperform TCP transfer, when applied in DVB-S systems. Recently, a novel LT-based codes are proposed in [11], specially designed for Earth-space channel. In [3] it was shown that Raptor codes could also be beneficial in blockage mitigation of Earth-space links.

The most powerful Raptor code, called RaptorQ fails to reconstruct K information symbols if it successively receives any $K + O$ codeword symbols with probability close to the failure probability of a random fountain code constructed over Galois field GF(256), which is upper bounded by $1/255 \times 256^{-O}$ [8]. Thus, if a receiver collects only a few additional symbols (for example $O = 4$), the achievable decoding failure becomes negligible compared to any practically required residual target codeword loss rate. It follows that for $K \gg O$ RaptorQ code performs closely to the MDS code, which can recover a transmitted codeword (with zero failure probability) if it collects only K codeword symbols. RaptorQ decoders are based on the inactivation decoding algorithm, which has a serial

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structure by nature, and does not represent a very effective solution for hardware implementation. In addition, memory sizes of the largest commercially available programmable chips are not sufficient to store a single Gb long codeword of a RaptorQ code, which may be required in some high data rate Earth-space links. Splitting and decoding a codeword across multiple hardware chips can be identified as the second major drawback to consider long RaptorQ codes for practical implementation.

To resolve drawbacks of long RaptorQ codes, we propose employment of interleaved Reed-Solomon (RS) codes. Multiple independent codecs run in parallel and connected to a single (or distributed across multiple chips) interleaver/deinterleaver, resulting in a design that is free of RaptorQ's drawbacks. However, such erasure protection scheme is applicable if size of the interleaver is close to the length of the optimal MDS code with the same code rate and residual loss rate. It should be noted that the interleaved RS codes are not necessarily MDS codes for arbitrary distributed erasure channel, and therefore, their applicability needs validation. In addition, performance evaluation of inverleaved RS codes by using Monte Carlo simulation is not practical, given the large interleaver sizes.

In this paper, we analyze Earth-space channels subjected to deep fades, which cause blockage of the channel. We assume that probability density of on/off states duration is modeled by Weibull distribution. First, we numerically obtain lower bounds on the propagation latency cased by employment of MDS protection code in considered fading channel, that achieves desired residual loss rate. In addition, we propose a simulation method that enables quick and accurate performance evaluation of interleaved RS codes. With aid of designed simulator we identify interleaved RS codes that perform closely to MDS codes on Earth-space channel. Surprisingly, for majority of code rates and channel parameters extremely short RS codes (of code lengths equal to 15) will provide satisfactory latency.

The rest of the paper is organized as follows. In Section II Earth-space channel model is given. In Section III a methodology for link reliability analysis is presented, while Section IV is dedicated to numerical results. Concluding remarks are given in Section V.

II. Earth-space channel modeling

The fade phenomenon in Earth-space links usually manifests though *fade episodes*, which are defined as time intervals in which signal attenuation due to fading exceeds a *fade threshold*. Similarly, as seen in Fig. 1, *inver-fade intervals* are complementary events, in which signal attenuation is below the fade threshold [13]. It is usually considered that fade episodes have catastrophical effect on transmission through Earth-space channel, i.e., information sent during fade episodes will be erased and will not appear at the receiver side. The information passes the communication channel only in inver-fade intervals. Possible information distortion during inver-fade intervals, due to signal strength variation and additive noise, is overcome by channel coding. If a channel code recovers all information in inver-fade intervals, recovering lost information during fade episodes is done by employing an additional packet erasure code. Through the paper, we will assume ideal information reconstruction during inver-fade intervals, meaning that probability of residual error after the channel coding is sufficiently low and the information reconstruction in fade episodes is not jeopardized by channel errors.

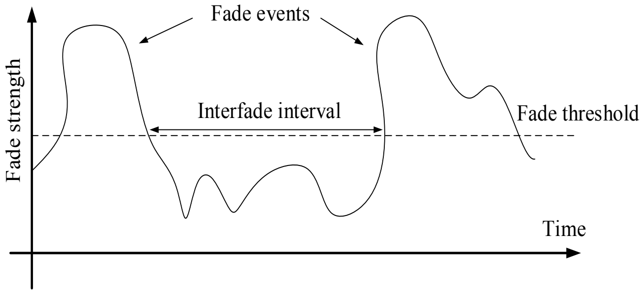

Fig. 1: Illustration of the fading model.

Under the aforementioned idealization, it is sufficient to describe the fading phenomenon through statistics of duration of fade episodes and inver-fade interval. Modeling such statistics, especially in Ka-band, was heavily investigated during the past years. Long-term measurements have revealed that fade episodes can be classified into two types, short-term episodes (usually less that 1 second) and long-term episodes (usually last significantly longer than 1 second), while each type has different probability density. The cause of long-term episodes is mostly rain, and average duration of long-term episodes is such that the link reliability will not benefit from deployment of packet erasure codes. Thus, will will focus only on short-term episodes, neglecting the effect of long-term fading to link reliability. We simply assume that the long-term fading is treated by some other way, for example through transmission power adaptation.

Specificity of Earth-space channel is its dependency on local climate, meaning that different points on the Earth surface will exhibit different short-term event statistics. Thus, it was proposed that in South Asia, North America and France fade episode duration is modeled with hyperexponential probability density function (PDF), while measurement in Brazil and Vancouver showed that Weibull distribution is a better fit [4]. On the contrary, measurements in Spain revealed that log-normal PDF is the most adequate to model short-term fading episodes [5]. Finally, ITU-R P.1623-1 recommends usage of the power-law distribution [13]. For the purpose of the numerical results, presented in Section IV, we will model fade episode duration as random variable following Weibull distribution. However, simulation mythology presented in Section III is invariant of the used statistical fading model.

Namely, we define the probability that a random fade duration d_f is longer than some D, given that the attenuation A exceeds a threshold a

$$P(d_f > D | a > A) = \exp(-(D/\lambda_f)^{k_f}),$$

where λ_f and k_f, represent shape and scale parameters, respectively. Similarly, we assume that interfade duration d_{if} follows the same probability low, i.e.,

$$P(d_{if} > D | a \leq A) = \exp(-(D/\lambda_{if})^{k_{if}}),$$

where λ_{if} and k_{if}, represent shape and scale parameters of the inver-fade distribution, respectively. The channel erasure probability is defined as

$$p = \frac{E[d_f]}{E[d_f] + E[d_{if}]} = \frac{T_f}{T_f + T_{nf}} = \frac{\lambda_f \Gamma(1 + 1/k_f)}{\lambda_f \Gamma(1 + 1/k_f) + \lambda_{if} \Gamma(1 + 1/k_{if})},$$

where $\Gamma(\cdot)$ represents gamma function and $E[\cdot]$ denotes expectation operator.

We further assume that d_f and d_{if} are mutually uncorrelated and that after each fade episode receiver performs acquisition, prolonging the start of an interfade interval by a fixed time period T_{acq}. The acquisition time is included in the reliability analysis presented in the following sections.

III. Link Reliability Analysis

Given the fact that Earth-space links exhibit large propagation delay, reliability of transmission can be increased only by employing codes that can be used to recover erased information. The optimal way to recover information is to use MDS codes, which can produce the lowest residual loss rate, compared to all other codes with the same length and code rate. Alternatively, among all other codes with length N an MDS code achieves desired level of residual loss rate, with the highest possible code rate $r = K/N$, where K denotes information length. Namely, based on the Singleton bound, MSD code can reconstruct a codeword if no more than $N - K$ code symbols are missing. Given the fact that transmission latency is proportional to code length, MDS codes provide bounding latency vs. code rate dependency, for the given fading channel statistics. We only consider the propagation latency which is equal to $L = N/R_s$, where R_s is the transmission bit rate.

Obtaining close form expression for minimal latency, assuming arbitrary fading distribution, is not feasible. Instead, we rely on simulation to provide bounding latencies for desired code rate, which are depicted in Section IV. Given the fact that average fade episode duration T_f is measured in milliseconds, minimal latencies must be higher than T_f. In addition, as it is considered that modern Earth-space links must provide rate measured in Gbps, MDS codes must be at least tens of megabits long. Clearly, only sub-optimal solutions are of practical interest, given the fact that decoding complexity of MDS codes grows quadratically with code length. A potential solution must include codes which decoding complexity is significantly lower. Here we investigate possibility of using interleaved MDS codes to provide satisfactory latency vs. code rate trade-offs. The main advantage of using interleaved MDS codes is that decoding complexity grows only linearly with the size of the interleaver (interleaver size represents code length). However, the unanswered question is how close inverleaved codes approach Singleton bound.

Consider transmission of packets with fixed size equal to D_P. Each packet is coded with an Reed-Solomon MDS code (N_c, K_c), with length N_c and code rate $r = K_c/N_c$. It is considered that binary length of the RS code $N_c \times \log_2(N_c) \ll D_P$, meaning that packets are divided and peace-by-peace coded. Before transmission codewords are passed through convolutional interleaver [14], which shuffles I codewords, distancing symbols from a same codeword by at least $I/(R_s * \log_2 N_c)$ seconds, where I represents the interleaver depth. At receiver side the deinterleaver restores the original bit order, while the decoder recovers the missing symbols. The propagation latency is defined as time required to fill the interleaver at the transmitter side and to empty deinterleaver at the receiver side, which based on the principe of convolution interleaving is equivalent to time needed to fill a symbol matrix of size $N_c \times I$, i.e., latency is equal to $L = \log_2(N_c) N_c I / R_s$. Given the fact that fade episodes usually corrupt multiple adjacent codewords, the packet loss rate (PLR) is approximately the same as codeword loss rate.

From the functional point of view the convolutional interleaver is the same as the block interleaver, which stores codewords in columns of $N_c \times I$ symbol matrix. By employing symbol-exact Monte Carlo simulations, it is straightforward to calculate the codeword loss probability of the decoder, i.e., the probability that the number of erased symbols in a any codeword is greater than $N_c - K_c$, for fixed K_c, N_c and I. Symbol-exact simulator alternately generates a large number of fade and non-fade blocks, according to assigned distributions, and memorize positions that correspond to start and end of each fading block. Unrepairable codewords are identified by searching and counting fade overlaps in each block interleaver column. Reliable symbol-exact simulators are computationally hungry since, for the fixed interleaver depth I and code length N_c, they process 10^8 interleaving blocks or higher with sizes measured in Gb (simulation of 10^8 interleaver blocks is needed to reliably estimate the target loss rate of 10^{-6}). Instead, we propose Segment-Based Simulator (SBS), which divides each interleaver row into S long segments, and estimates decoder failure probability assuming that codewords inside a segment are either all correctly reconstructed or all erased. This means that SBS gives an upper bound for the codeword loss rate (CWLR), for an arbitrary chosen RS codes. Inputs to the algorithm are interleaver depth measured in segments $I_{seg} = I/S$ and the number of segments that need to be simulated ($NumSegments$). We formally express SBS using algorithmic notation, as depicted in Algorithm 1.

If segment length is equal $S = 1$ symbol the SBS becomes the symbol-exact simulator. In a limiting case, for sufficiently large interleaver depth, the channel is transformed into symbol erasure channel with loss rate of

$$CWLR = \sum_{i=N_c-K_c+1} \binom{N_c}{i} P^i P^{N_c - i},$$

where P represents the erasure rate of channel that includes acquisition time, i.e,

$$P = \frac{T_f + T_{acq}}{T_f + T_{nf}}.$$

As an illustration, in Fig. 2, we depict dependency of CWLR on the introduced latency, by using (15,11) RS code. With slice modification SBS can be used to determine

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Algorithm 1: Segment-Based Simulator

Input: I_{seg}, $NumSegments$

Output: $CWLR = errorCount/SegmentCount$

$SegmentCount \leftarrow 0$

$errorCount \leftarrow 0$

while $SegmentCount < I_{seg} \times N_c \times NumSeg$ **do**

  $X = (X_1, X_2, \ldots, X_i, \ldots, X_t), \quad t \gg 1, \quad X_i \sim Weibull(\lambda_{nf}, k_{nf})$

  $Y = (Y_1, Y_2, \ldots, Y_i, \ldots, Y_t), \quad t \gg 1, \quad Y_i \sim Weibull(\lambda_f, k_f)$

  $F = (F_1, F_2, \ldots, F_{2i}, F_{2i+1}, \ldots, F_{2t}), \quad F_j = \begin{cases} \lceil (Y_i + T_{acq}) R_s / S \rceil, & j = 2i \\ \lceil (X_i - T_{acq}) R_s / S \rceil, & j = 2i - 1 \end{cases}$

  $S_i = [a_i, b_i] : \ a_i = 1 + \sum_{j=1}^{2i-1} F_j, \ b_i = a_i + F_{2i} - 1$

  $j \leftarrow 0$

  while $j < \lfloor b_t / (N_c \times I_{seg}) \rfloor$ **do**

    $i \leftarrow 0$

    $j \leftarrow j + 1$

    while $i < I_{seg}$ **do**

      $i \leftarrow i + 1$

      $m \leftarrow i + (j - 1) \times N \times I_{seg}$

      $M = \{m, m + I_{seg}, \ldots, m + (N - 1) \times I_{seg}\}$

      $E = \{m_q | m_q \in M \wedge m_q \in \{S_1 \cup S_2 \ldots \cup S_n\}\}$

      if $|E| > N_c - K_c$ **then**

        $errorCount \leftarrow errorCount + 1$

  $TotalSegmentCount \leftarrow TotalSegmentCount + I_{seg} \times j$

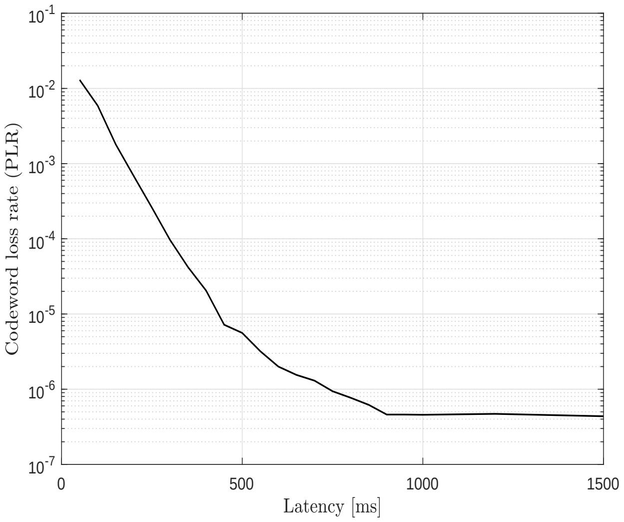

Fig. 2: PLR as a function of latency for (15,11) code ($T_f = 10$ ms, $p = 0.01$ and $k_f = k_{nf} = 1$, $S = 100$ KSymbol).

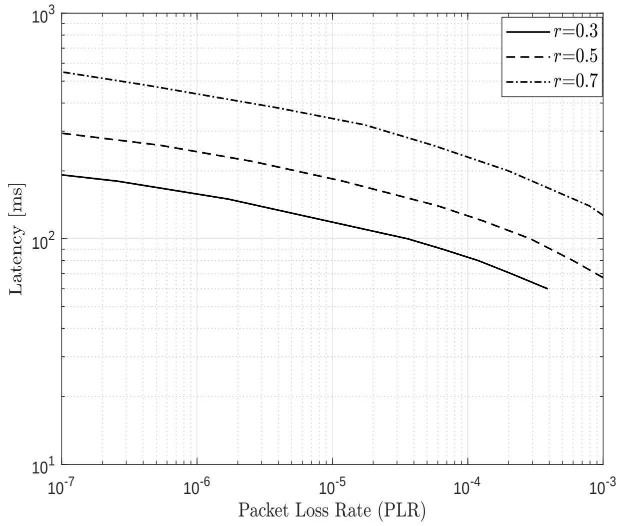

Fig. 3: Latency lower bounds for different PLR values and code rates ($T_f = 10$ ms, $p = 0.01$ and $k_f = k_{nf} = 1$).

a bounding latency for MDS codes also, when code length of a code is equal to size of the interleaver.

Clearly, speed up of SBS simulator, compared to the symbol-exact Monte Carlo simulator is proportional to segment length S. In the following section we use $S = 100$ Ksymbol long segments, for $R_s = 10$ Gbps links to show that in SBS can adequately evaluate code performance. This means that SBS is roughly five orders of magnitude faster than symbol-exact simulator.

IV. Numerical Results

We first, based on the methodology presented in Sections II and III, provide lower propagation latencies required to achieve desired PLR. In all presented results in this section we assume $D_P = 1$ Kb and $R_s = 10$ Gbps. In Fig. 3 required latency dependence of PLR is depicted, for various code rates, assuming $T_f = 10$ ms, $p = 0.01$ and $k_f = k_{nf} = 1$. For example, if $PLR = 10^{-6}$, it follows that it is not possible to construct protection code that introduces latency lower than $L = 250$ ms with code rate $r = 0.5$. If communication service requires lower

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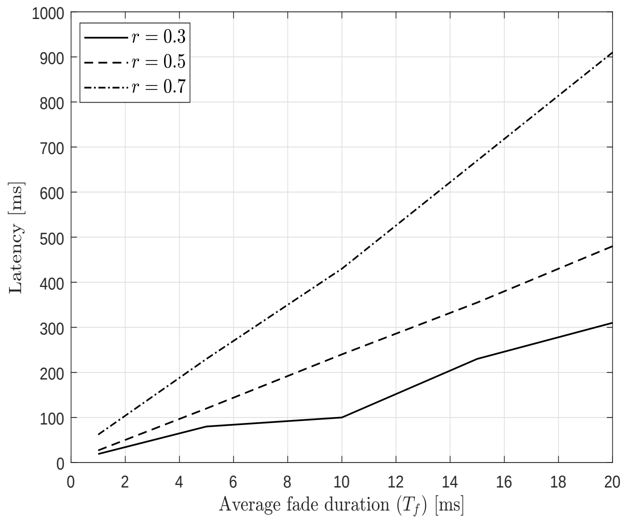

Fig. 4: Latency lower bounds as function of T_f ($PLR = 10^{-6}$, $p = 0.01$ and $k_f = k_{nf} = 1$).

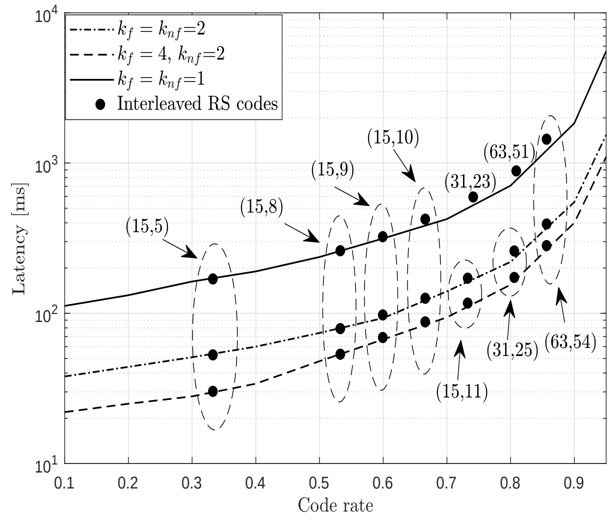

Fig. 5: Latency lower bounds as function of T_f ($PLR = 10^{-6}$, $p = 0.01$ and $k_f = k_{nf} = 1$, $S = 100$ Ksymbol).

latency spatial diversity can be used. For example, voice communications cannot successfully operate with latencies above 50 ms, which means that to operate with code rate $r = 0.5$ and $PLR = 10^{-6}$, five spatially distant links need to be established. It should also be emphasized fundamental trade-off between latency and achievable code rate, i.e., it is not possible to simultaneously increase code rate and reduce bounding latency. In Fig. 4 we depict bounding latencies for different average fade episode durations. We can observe clear linear latency growth, while slope is dependent of a considered code rate.

Finally, in Fig. 5 we investigate latencies achievable with interleaved RS codes, for the fixed $PLR = 10^{-6}$. From implementation perspective RS codes should be as short as possible. We see that it is possible to perform at lower latency bound with code length $N_c = 15$ for low and medium code rates, while code lengths of $N_c = 31$ and $N_c = 63$ are sufficient to operate at low latency bound for higher code rate. As channel condition deteriorate, which is expressed through change of parameters k_f and k_{nf} higher code length are required. For example, for $k_f = k_{nf} = 2$ it is sufficient to use (15,11) RS code to operate at latency bound, while for $k_f = k_{nf} = 1$ at least (31,23) is needed.

V. Conclusion

In this paper we provided a new look to the problem of reliable transmission through Earth-space channels, under deep fade phenomenon. We have shown that short RS coupled with convolutional interleaver ensure near optimal performance in terms of propagation latency, which cannot be avoided if we aim to ensure desired level of residual error loss rate. Interestingly, for large number of code rates it is sufficient to employ short RS codes, of length 15, which represents a low complexity solution.

During further research will use results in presented in this paper, to propose HST system, with increased information capacity, in which handover operations have extremely low blockage probability, which is currently an open research problem.

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