Effect of Phase Noise on Error Probability of MPSK Receiver over TWDP Channel - Simulation Study

Goran T. Djordjevic, *Member, IEEE*, Jarosław Makal, Bata Vasic and Bane Vasic, *Fellow, IEEE*

***Abstract*— The estimation of multilevel phase-shift keying (MPSK) signal phase is adversely affected by the number of factors appearing in transmission channel and related to the receiver. The imperfect reference signal phase estimation causes error probability degradations. The higher the value of frequency, the stronger is the effect of imperfect reference signal phase recovery. The aim of this work is to study the effect of imperfect phase estimation on error probability when signal propagates through a channel where there are two dominant components and a diffuse component, which could be the model for mmWave range. The Two-Wave Diffuse-Power (TWDP) model can accurately characterize this propagation environment. We develop a simulation model to estimate the error floor value and to identify the range of signal power when floor appears. Our simulation results give direct dependence of error rate value on channel parameters, signal power and standard deviation of phase noise. The aim of the paper is not to present the concrete estimator architecture, but to make a further step in studying the effect of a certain phase noise on bit error rate performance.**

***Index Terms*— Error probability, fading channel, phase noise, Monte Carlo simulations.**

I. Introduction

DURING the signal propagation through a wireless telecommunication channel, random changes of the signal parameters occur at the detection point. The signal level is no longer constant, but changes randomly, and the signal phase is a random process. These changes in signal level and phase have a significant impact on the quality of information transmission through that channel. Depending on the weather conditions and the type of the propagation environment including the receiver surrounding, different models have been developed to describe variations in signal intensity [1].

To describe multipath fading, different models of this fading have been proposed, such as Rayleigh, Rice, Nakagami-*m*, Nakagami-*q*, etc. [1]. In the case when two direct waves reach the receiver, but there is also the diffuse component of the emitted EM wave, the so-called Two-Way Diffuse-Power (TWDP) model of fading was proposed in [2]. In that paper in which the TWDP model was initially suggested, an approximation was proposed for the probability density function (PDF) of the signal envelope variations. This model of fading has been constantly the subject of study by many researchers [3]-[9]. In [3], Kim *at all* emphasized some shortcomings of approximation from [2], and they presented exact and approximate formulae for bit error rate in detecting binary phase-shift keying (BPSK) signal transmitted over a TWDP channel for large values of average signal-to-noise ratios (SNR). In [4] and [5], the authors derived some novel expressions for system performance metrics and derived interesting result showing that TWDP fading model has closed form moment generation function (MGF) of received signal envelope. The authors of [6] presented a novel way of parametrization of TWDP channel model. Generally, characterization of TWDP model is in tight relation with propagation environment where there are useful signal, co-channel interference and additive white Gaussian noise [10].

Recently it has been shown experimentally that this model of fading is just appropriate for describing signal propagation in the 60 GHz range [7]-[9].

In all those papers, it is implicitly assumed that the estimation of the phase of the received signal is ideal. However, it is well known that there is a difference between the phase of the incoming signal and the signal phase at the phase estimator output. This difference is a random process that affects signal detection. Due to the existence of this difference, the performance of the system deteriorates. The influence of this phase noise on the error probability when multipath fading is modeled by Nakagami-*m* distribution was considered in [11], while the effect of imperfect reference signal extraction on the error probability over a shadowed multipath fading was analyzed in [12]. In both papers, it was illustrated that error performance of a system can be strongly influenced by imperfect reference signal recovery. With the transition to higher frequency ranges, the influence of the phase noise becomes more pronounced. To the best of our knowledge, the impact of the phase noise on coherent detection in TWDP channel has not been considered so far. However, there is a significant difference between propagation environment considered in [11], [12], where one dominant direct component exist, compared with a TWDP channel model where two dominant waves exist. Our aim in this paper is not to deal with a concrete architecture of phase estimator, but to make a further step in studying the performance of TWDP coherent systems. We suppose the

Goran T. Djordjevic and Bata Vasic are with Faculty of Electronic Engineering, University of Nis, 14 A. Medevedeva, 18000 Nis, Serbia (e-mail: goran@elfak.ni.ac.rs; bata.vasic@elfak.ni.ac.rs)

Jarosław Makal is with Faculty of Electrical Engineering, Białystok University of Technology, Wiejska 45D street, 15-351 Białystok, Poland (e-mail: j.makal@pb.edu.pl).

Bane Vasic is with the Department of Electrical & Computer Engineering, University of Arizona, 1230 E. Speedway Blvd. P.O. Box 210104, Tucson, AZ 85721-0104, USA, (e-mail: vasic@ece.arizona.edu).

 ISBN 978-86-7466-930-3

phase error in the receiver has widely accepted Tikhonov distribution and determine the influence of the phase noise on the bit error rate (BER) performance of the receiver of multilevel phase-shift keying (MPSK) signals transmitted over a TWDP channel. We develop appropriate simulation model and present some Monte Carlo simulations results illustrating the effect of phase noise on the BER performance in detection of MPSK signals. These results obtained under assumption of the presence of phase noise can be compared with the results previously reported in [2]-[6], where the perfect reference signal estimation was considered. The design of phase estimator over a TWDP channel will stay an open problem, but relation between BER, channel parameters and standard deviation of phase noise will be a useful standpoint in design of an estimator.

II. System model

In this Section, we describe briefly the model of the system considered here. We give basic information about modulation/demodulation process, as well as channel characteristics.

At the transmitter, signal is modulated by performing multilevel phase-shift keying. During the duration of one symbol, the modulator output signal has the form $s = Ae^{j\phi_n}$, where

$$\phi_n \in \left[0, 2\pi/M, ..., 2(M-1)\pi/M\right], \tag{1}$$

where M is the number of phase levels.

This signal is transmitted over a wireless channel. During signal transmission, many copies of the transmitted EM wave excite the receiver antenna. The resulting signal envelope consists of specular and diffuse parts. Specular part contains two direct components having constant amplitudes and uniformly distributed phases in the interval from 0 to 2π. The amplitudes of the direct components are denoted by V_1 and V_2, while the phases of these components are denoted by ψ_1 and ψ_2. The scattering component has Rayleigh distribution, i.e., it consist of the in-phase and quadrature components having Gaussian distribution with zero mean value and standard deviation denoted by σ_F. These in-phase and quadrature components are denoted by x_F and y_F, respectively. The resulting received signal envelope is presented as [2]

$$r = V_1 e^{j\psi_1} + V_2 e^{j\psi_2} + x_F + jy_F \,. \tag{2}$$

This model of fading can be described in terms of two parameters denoted by K and Δ. The parameter K denotes the power of the specular components-to-power of the diffuse component. The parameter Δ is related to the ratio of the peak specular components power-to-average specular components power. These two parameters are defined as [2]

$$K = \left(V_1^2 + V_2^2\right) / \left(2\sigma_F{}^2\right), \tag{3}$$

$$\Delta = \frac{\text{Peak Specular Power}}{\text{Average Specular Power}} - 1 = \frac{2V_1V_2}{V_1^2 + V_2^2}\,. \tag{4}$$

The largest the value of the parameter K, the power of the specular components is larger compared with the power of the diffuse component. In other words, fading is shallower. The typical values of parameter K for the terrestrial mobile links are in the range from 0 dB to 15 dB. The values of parameter Δ lie in the range from 0 to 1. When Δ is equal to 1, specular components have the equal amplitudes, while when Δ is equal to zero either specular components amplitude is equal to zero.

It is interesting to note that the current envelope values of the signal propagating through the channel cannot be described using PDF in closed form. One approximation of the PDF was proposed in [2], while a PDF in infinite series form was recently proposed [6]. On the other hand, although there is no exact closed-form solution for PDF, there is a closed-form MGF format for this channel [4], [5]. However, for Monte Carlo simulations, the most important is eq. (2) defining the composite fading signal envelope.

After signal transmission over a TWDP channel, the receiver input signal can be presented as

$$y = re^{j(\phi_n + \psi)} + n\,, \tag{5}$$

where ψ denotes signal phase due to multipath propagation, and n denotes the thermal noise. This thermal noise can be described by Gaussian PDF with zero mean value and standard deviation σ.

In the case of BPSK, the signal in the receiver has the form

$$z = \pm r\cos(\varphi) + x \,, \tag{6}$$

depending upon "1" or "0" is transmitted. In the case of MPSK, the signal in the upper and down branch of the receiver [13, p. 361] is given, respectively, by

$$z_u = r\cos(\phi_n + \varphi) + x\,, \quad z_d = r\sin(\phi_n + \varphi) + y\,. \tag{7}$$

In the previous equations, r denotes the signal envelope defined by (2), ϕ_n denotes the modulated signal phase, φ denotes the phase noise, while x and y are the in-phase and quadrature thermal noise components over a channel. The signal detection is performed based on the value of the angle γ_d that is defined by $\tan\gamma_d = z_d / z_g$.

On the contrast to previous papers considering signal transmission over TWDP channel, in our paper the emphasis is on the imperfect reference signal recovery. Actually, the estimation of the received signal phase is not just perfect and there is a difference between received signal phase and estimated signal phase performed by an estimator. This difference is a random process. The values of this process are denoted by φ. In [14], it was shown that in the case when there is a signal and additive noise at the receiver input, then the phase difference has a Tikhonov distribution. Also, it has been shown that when a useful signal is influenced by fading in such a way that there is one regular component, the Tikhonov distribution is a satisfactory model for the phase error. However, in the case where there is a modulated signal with two regular components (TWDP channel), the phase estimation process is more complicated. The design of the phase estimator itself is beyond the scope of this paper. In the first, rough step, we will consider that the phase estimation is not ideal and that the phase difference has a rough approximation of the Tikhonov distribution. We will establish a connection between the standard deviation of phase noise, channel parameters and signal power. However, the question remains as to how the standard deviation of the phase error is related to the receiver and fading parameters.

The random variable having uniform distribution can be generated according to the algorithm presented in [15, p. 340], and random variable having Gaussian distribution can be generated using Box-Muller method [16, p. 383]. The random values of the signal envelope are generated taking into account eq. (1). The random variables having Tikhonov distribution can be generated by applying the Modified acceptance/rejection method [16, p. 382].

III. Numerical Results

In this Section, we present simulation results and give appropriate comments. The simulation results are obtained by Monte Carlo simulations performed in C++. The maximum number of generated samples for estimating a value of BER is 2×10^9. The methods utilized for generation of samples are described in the previous Section.

Fig. 1 illustrates the dependence of the BER on the average signal-to-noise power ratio (SNR), γ_b. The BER decreases with increasing SNR. However, this decrease in the BER is not uniform in the whole range of SNRs, but is expressed for low and moderate values of SNRs. In the range of high values of the SNR, the BER value does not decrease with increasing SNR. In other words, the BER tends to a constant value called the error floor. The appearance of this floor is a direct consequence of the presence of phase noise, i.e., non-ideal extraction of the reference carrier. The value of the floor depends on the standard deviation of the phase noise. The higher the standard deviation of the phase noise, the lower the value of the floor. This value of the floor cannot be reduced by increasing the signal power, but it can only be influenced by the correct design of the part of the system in which the phase of the received signal is estimated.

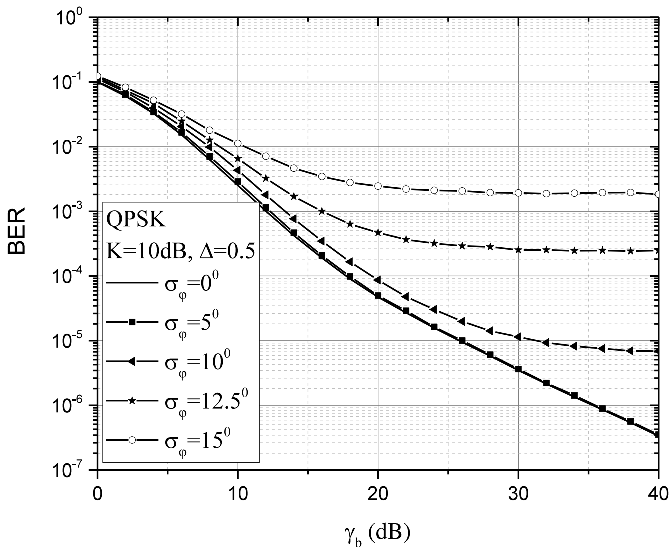

Fig. 1. BER performance for different values of standard deviation of phase noise.

Fig. 2 shows the influence of the standard deviation of the phase noise on the BER values in the detection of BPSK and QPSK signals. Firstly, it is obvious that BER values increase with increasing standard deviation values. Secondly, with the QPSK format, the BER value remains unchanged in the range of standard deviation values up to 8 degrees, while the BPSK format is insensitive to changing the standard deviation up to 16 degrees. In other words, the BPSK format is more resistant to the influence of the phase noise. This result is also logical because the areas of decision-making in the BPSK format are wider, so that the phase noise has less possibility to move the point from one area of decision-making to another.

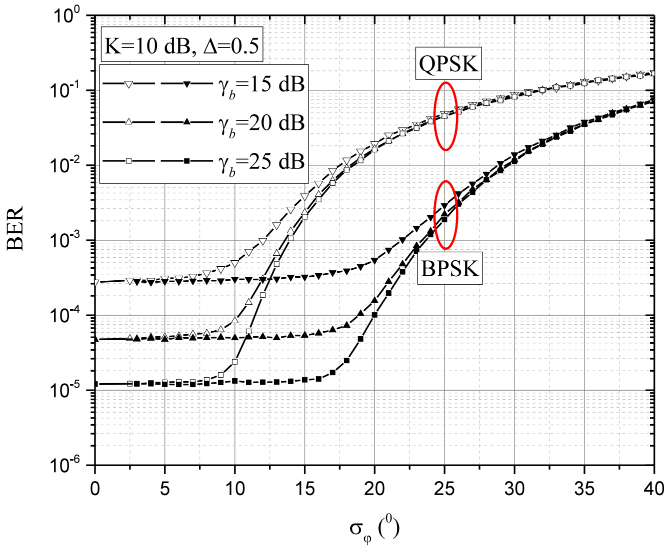

Fig. 2. BER performance for BPSK and QPSK modulation formats.

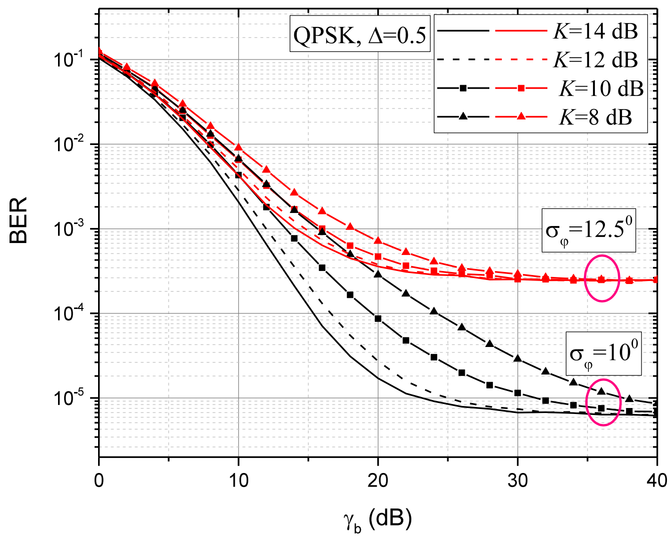

Fig. 3. BER performance for different values of fading parameter *K*.

The influence of the ratio of the average powers of the direct and scattered part of the resulting signal (parameter K) is shown in Fig. 3. The influence of the parameter K on the BER value is pronounced in the range of moderate values of the SNR and in the case when the standard deviation of the phase noise is smaller. In the region of large values of the SNR, BER tends to a constant value regardless of the value of the parameter K. This parameter has no effect on the value of the BER floor. In addition, when the value of the standard deviation is higher, then the influence of the phase noise is

 ISBN 978-86-7466-930-3

more pronounced, so the influence of the parameter K is non-dominant.

Similar conclusions as in the previous case can be drawn regarding the influence of the parameter Δ on the BER value. Namely, the parameter Δ also has a significant effect on the BER values in the middle range of the SNR, while there is no effect on the BER values when the signal power is high, i.e., when the BER floor is reached. Also, when the standard deviation of the phase noise is larger, i.e., when the influence of non-ideal extraction is dominant, BER values are less sensitive to changes in parameter Δ compared to the case when the extraction of the reference signal phase is more precise.

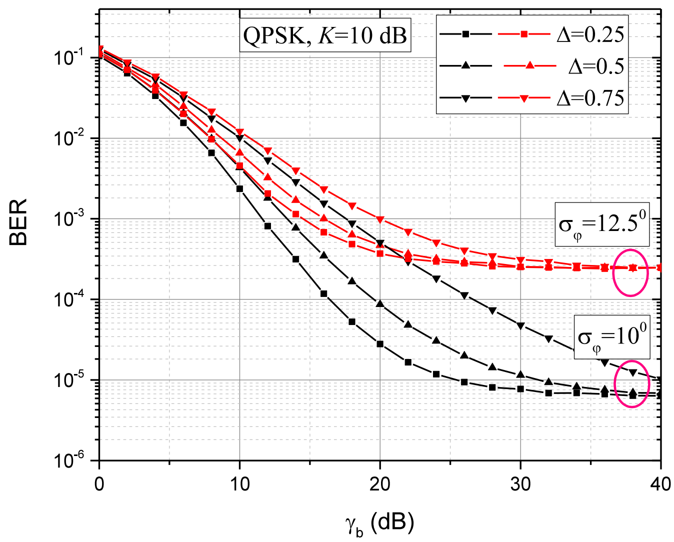

Fig. 4. BER performance for different values of fading parameterΔ.

IV. Conclusion

We have analyzed the influence of non-ideal extraction of the reference carrier on the BER performance of the receiver of MPSK signals propagating through a channel in which variations in signal levels are characterized using the TWDP model. It can be concluded that imperfect estimation of the phase of the incoming signal causes the appearance of BER floor. The value of this BER floor can be reduced by decreasing the value of the standard deviation of the phase noise, i.e., by improving the received signal phase estimation. The influence of the parameters K and Δ is pronounced in the range of moderate SNR values. In addition, the results have shown that for typical values of the channel parameters, BPSK modulation format is resistant to phase noise up to the standard deviation value of 16 degrees, while in the same environment QPSK format is resistant to this phenomenon only up to the standard deviation value up to 8 degrees. The design of the phase estimator should be performed so that the predetermined value of the standard deviation of the phase noise is not exceeded.

The results obtained here should be considered as the most optimistic when there is a phase error. The problems of loop locking and phase skipping in the channel with TWDP fading remain open, as well as the connection between the standard deviation of the phase noise and the estimator parameters. Incorporation of some innovative phase noise mitigation techniques, like Viterbi-Viterbi noise estimator under given channel scenario also remains an open problem.

Acknowledgment

This work is partially supported by The Polish National Agency for Academic Exchange (NAWA) under grant No. PPN/ULM/2020/1/00256/DEC/1, and partially supported by the Science Fund of the Republic of Serbia, grant No. 7750284, *Hybrid Integrated Satellite and Terrestrial Access Network - hi-STAR*, as well as by Ministry of Education, Science and Technological Development of the Republic of Serbia.

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