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A Numerical Method for Estimating Error Rate Performance of MPSK System influenced by Fisher-Snedecor Fading

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***Abstract*—The aim of this paper is to estimate the error probability in coherent detection of multilevel phase-shift keying (MPSK) signals transmitted through a channel in which the multipath fading and shadowing are present simultaneously together with thermal noise. To describe this type of fading, we use a Fisher-Snedecor model that was proposed relatively recently. For this fading model, using the Fourier series, we first represent the probability density function (PDF) of the composite signal phase. We present expressions for the coefficients in the Fourier series, that were previously derived by ourselves. We examine the convergence of this series for different numerical values of the channel parameters. Next, we present the expression for estimating the error probability in MPSK signal detection. Based on this, we examine the influence of multipath fading severity and the shadowing spread on the numerical values of the error probability for different number of phase levels. The obtained results enable a further evaluation of the performance of differentially coded signals, as well as an evaluation of the influence of the phase noise both on the error probability and the mutual information in the channel with this type of fading.**

Keywords—error probability, fading, Fourier series, probability density function.

I. INTRODUCTION

One of the important parameters of the communication system is an error probability when detecting a signal conveying information. This error probability can be determined in different ways: by measurement, simulations or analytical and numerical methods. In order to perform measurements, it is necessary to realize the entire system physically and to have the equipment for measurement. Monte-Carlo simulations are generally time-consuming and to a large extent the confidence interval of estimation of error probability depends on the developed simulation model. Analytical/numerical approach is suitable because the evaluation is done on the basis of derived formulas and the influence of the channel parameters on the value of the error probability itself can be easily observed [1], [2].

One of the analytical methods for estimating the error probability is based on the knowledge of the probability density function (PDF) of the phase of the composite signal propagating through the channel. If this PDF of phase is known, it is necessary to take only one more step to estimate the probability of error - the integration of the obtained PDF within certain limits. It is possible to expand the PDF of the signal phase into a Fourier series. This method is convenient because in that case the integration of the PDF is a trivial operation. In the case when the signal propagates through the channel with thermal noise, the derivation of the expression for the PDF of signal phase is shown in [3], [4]. In the case when the signal propagates through a channel with multipath fading, the PDF of the phase is derived in [5]. Apart from multipath fading, a shadow effect is usually present in the channel at the same time. In the case when the channel in which multipath fading and the shadow effect are simultaneously present the propagation environment is modeled using the generalized-K distribution. In that case, the derivation of the PDF of phase of the signal is presented in [6]. An estimate of the truncation error during the numerical calculation and some other details related to the numerical calculation in this case are given in [7].

Recently, a new fading model was presented in the literature, which is suitable for describing signal propagation through a channel in which multipath fading and the shadow effect are present. This model is known as the Fisher-Snedecor fading model [8]. The measurements confirmed that this model is especially useful for device-to-device (D2D) communications [8]. In this paper, our goal is to show the Fourier series representation of the PDF of composite signal phase for this fading model and give the analytical forms coefficients in this Fourier series. In [9], new expressions for the coefficients in the Fourier development of the PDF were derived. However, there

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is still room to investigate how many terms of the series are sufficient to reach the given accuracy in the numerical calculation of the PDF of the composite signal phase. In this paper, the emphasis is precisely on this assessment, because other calculations necessary for assessing the quality of information transmission through the telecommunications channel also depend on the PDF of the phase. In addition, some novel insights should be presented into error performance when coherent detection of phase-shift keying (PSK) signals is performed. Further, we present the expressions for estimating the error probability in which the previously derived Fourier series coefficients are present. Based on these expressions, we investigate the influence of the fading severity and the shadow effect on the values of the probability of error in PSK signal coherent detection.

The paper is organized in the following way. In Section II, we give the Fourier series form of the PDF of composite signal phase and the equations for estimating the error probability. In Section III, we estimate the required number of terms in order to reach the given accuracy in estimating PDF of composite signal phase. Section IV is reserved for some numerical results followed by discussions. Section V presents concluding remarks and plans for further research.

II. Analytical formulas for PDF of phase and symbol error probability

In this Section, we briefly introduce the channel model and present analititical expressions for coefficients in Fourier form of PDF of composite signal phase valid for channel with thermal noise and shadowed multipath fading, as well as expressions for symbol error rate (SER).

A. Channel model

Based on the fading model proposed in [8], the PDF of envelope of a signal propagating through a channel with Fisher-Snedecor fading is described by

$$f_R(r) = \frac{2m^m (m_s\Omega)^{m_s} r^{2m-1}}{B(m,m_s)(mr^2+m_s\Omega)^{m+m_s}}, \tag{1}$$

where $\Omega = \mathrm{E}\left[r^2\right]$, while m and m_s are parameters defining the severity of multipath fading and shadowing, respectively. The larger the value of parameter m, the shallower is the multipath fading. Similarly, the larger the value of parameter m_s, the shallower is the shadowing effect. In this work, we assume without lossing generality that $\Omega = 1$.

Starting from (1), it can be easily shown that PDF of instantenous signal-to-noise ratio (SNR) is given by

$$f_\gamma(\gamma) = \frac{m^m (m_s\bar{\gamma})^{m_s} \gamma^{m-1}}{B(m,m_s)(m\gamma+m_s\bar{\gamma})^{m+m_s}}, \tag{2}$$

where $\bar{\gamma} = \mathrm{E}[\gamma]$ is average SNR.

The PSK signal is transmitted over the channel where instantneous SNR can be described by (2). At the receiver, the decision is made on the basis of the value of phase value at the sampling moment.

B. PDF of composite signal phase

The PDF of the composite signal phase, ψ, can be described by Fourier series given by [7]

$$f_\psi(\psi/\gamma) = \frac{1}{2\pi} + \sum_{n=1}^{\infty} a_n(\gamma)\cos(n\psi), \quad |\psi| \le \pi, \tag{3}$$

where coefficients $a_n(\gamma)$ depends on instantenous SNR and they are given by [7]

$$a_n(\gamma) = \frac{1}{n!\pi}\Gamma\left(\frac{n}{2}+1\right)\gamma^{\frac{n}{2}}\exp(-\gamma)\,{}_1F_1\left(\frac{n}{2}+1;n+1;\gamma\right). \tag{4}$$

In order to emphasize the instantaneous SNR dependence, in (3) the PDF of composite signal phase is denoted by $f_\psi(\psi/\gamma)$. The average PDF of composite signal phase is given by

$$f_\psi(\psi) = \int_0^\infty f_\psi(\psi/\gamma) f_\gamma(\gamma)\,\mathrm{d}\gamma \ . \tag{5}$$

By combining (2), (3), (4) and (5), with help of several identities from [10], the average PDF of composite signal phase has the form of

$$f_\psi(\psi) = \frac{1}{2\pi} + \sum_{n=1}^{\infty} b_n\cos(n\psi), \quad |\psi| \le \pi, \tag{6}$$

where coefficient b_n is given by

$$b_n = \frac{n}{2\pi\Gamma(m)\Gamma(m_s)} G_{2,3}^{2,2}\left(\frac{m_s\bar{\gamma}}{m}\left|\begin{matrix}1-m,1\\ m_s,\frac{n}{2},-\frac{n}{2}\end{matrix}\right.\right), \tag{7}$$

where $G_{p,q}^{m,n}(.)$ denotes special Meijer's G function [11, eq. (9.301)].

Detailed derivation can be found in [9].

In this way, the PDF of composite signal phase is represented in the form of Fourier series whose coefficients are expressed in analytical form. However, the PDF is given in the form of the infinite series and summation should be terminated after a finite number of terms in order to obtain a numerical value of PDF.

C. Error probability

The symbol error probability in detecting MPSK signal transmitted over a channel with Fisher-Snedecor fading can be estimated by integration of (6) over the range of phase values that do not correspond to given signal phase decision region

$$P_e = 1 - \int_{-\pi/M}^{\pi/M} f_\psi(\psi)\,d\psi \ . \tag{8}$$

After elementary integration, it follows the expression for symbol error probability given in the form of infinite series given by

$$P_e = 1 - \frac{1}{M} - \sum_{n=1}^{\infty} \frac{2b_n}{n} \sin\frac{n\pi}{M} \quad (9).$$

III. Required number of terms in PDF series (6)

If we denote

$$S_N = \sum_{n=1}^{N} b_n \cos(n\psi) \quad (10)$$

in (6), we perform summation until the condition

$$|S_N - S_{N-1}| \le \varepsilon \quad (11)$$

is satisfied, where $\varepsilon = 10^{-6}$ is given accuracy.

Fig. 1 shows the required number of series terms in (6) to achieve an accuracy of 10^{-6}. It can be concluded that the required number of terms increases in the area of higher average SNR values.

Besides Fig. 1, Table I also illustrates the required number of terms under condition of achieving given accuracy. It can be concluded that the given accuracy is achieved with the number of terms smaller than 750, which nowadays is not a problem to realize on personal computers. Also, the number of terms is the largest for phase values of π/n, $n \in Z$ and at the value of phase equal to zero.

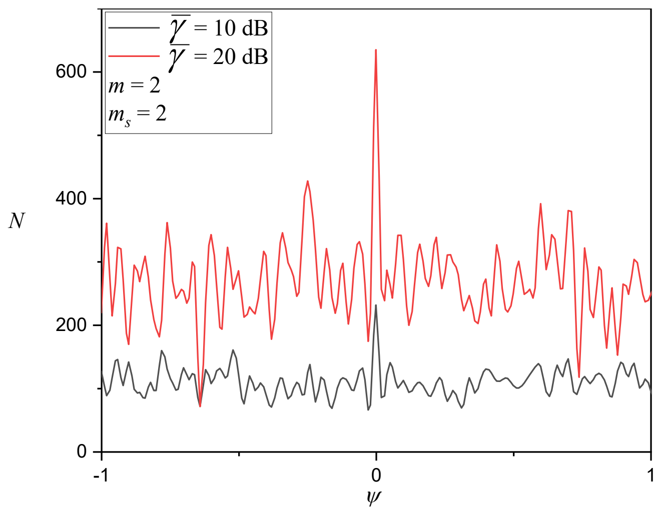


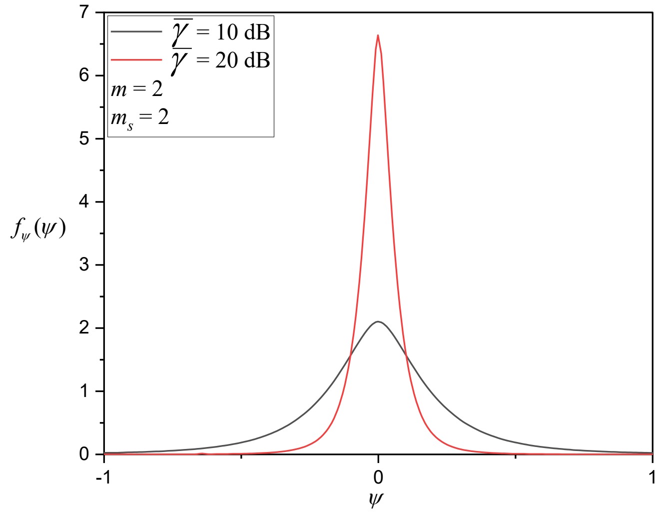
Fig. 1. Required number of terms in (6) for different values of average SNR.

TABLE I. Number of terms in summation of (6)

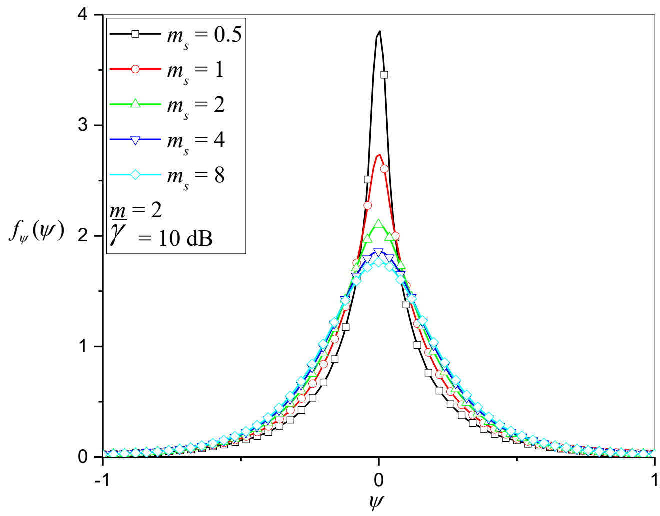
	Number of terms, $m = 2$, $m_s = 2$		
	$\overline{\gamma}$ = 10 dB	$\overline{\gamma}$ = 15 dB	$\overline{\gamma}$ = 20 dB
$\psi = 0$	236	419	744
$\psi = \pi/2$	237	419	745
$\psi = \pi$	236	419	744

IV. Numerical results

In this Section, we show some numerical results obtained by applying formulas (9) and (7). For all numerical values it was necessary to use less than 1000 terms in summation in (9). The required number of terms increases with the increasing the value of $\overline{\gamma_b} = \overline{\gamma} / \log_2 M$.



(a)



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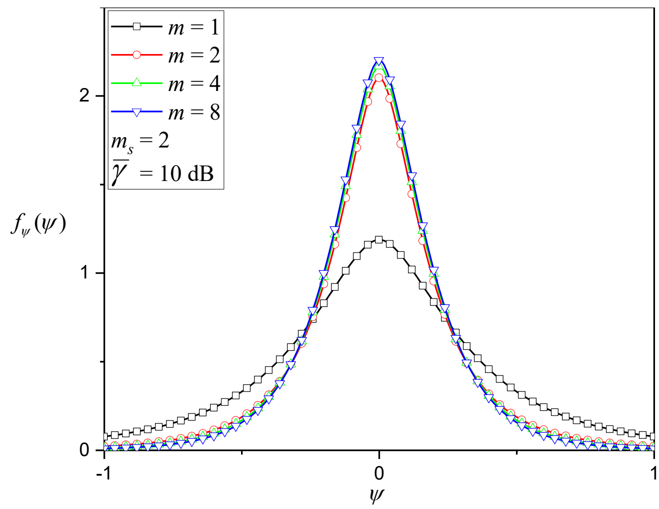

(c)

Fig. 2. PDF of signal phase for different channel parameters

Fig. 2 shows the shape of the PDF of the composite signal phase. Taking into account that the error probability is determined by integrating this PDF over some interval, the shape of the PDF is very important. It is especially important that the tails of the PDF are calculated correctly. Figure 2(a) shows the influence of the average SNR on the shape of the PDF, while Figures 2(b) and (c) illustrate the influence of the shadowing and multipath fading parameters on the PDF curve. We conclude that the curve is narrower and has larger peak if the average SNR value is higher, i.e., in a situation where multipath fading is shallower and shadowing is less pronounced. Therefore, the influence of these parameters on the probability of error can be immediately predicted.

Fig. 3 shows the dependence of SER for different values of the number of phase levels. As expected, for a fixed value of average SNR, the error probability increases as the number of levels increases. Let's say, for $\overline{\gamma}_b$ =15dB, the error probability increases 44.7 times when the number of levels increases from $M = 2$ to $M = 16$.

The influence of the multipath fading severity on the error probability is illustrated in Fig. 4. It has already been said that the fading depth is larger if the value of the parameter m is smaller. If the average SNR is fixed at 15 dB, the error probability decreases from 1.3×10^{-2} to 1.7×10^{-4} times when the parameter m increases from 1 to 4 (fading severity decreases).

The effect of shadowing spread on error probability is visible in Fig. 5. It can be concluded that for these parameters, the effect of shadowing spread on error probability is not so strong like the effect of multipath fading severity.

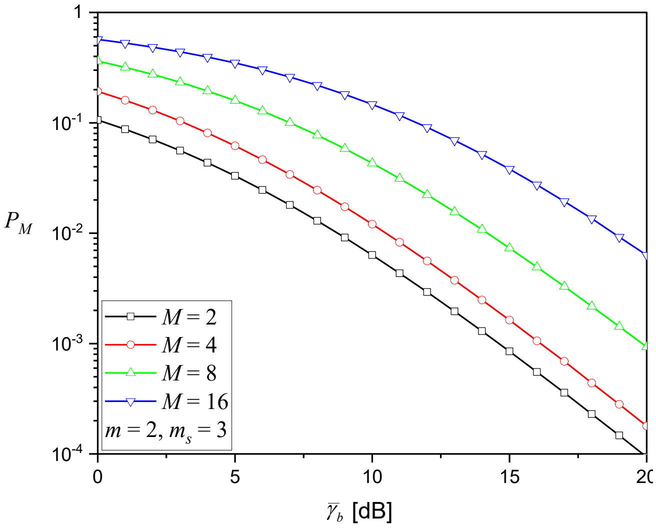

Fig. 3. SER perfomrance for different values of phase levels.

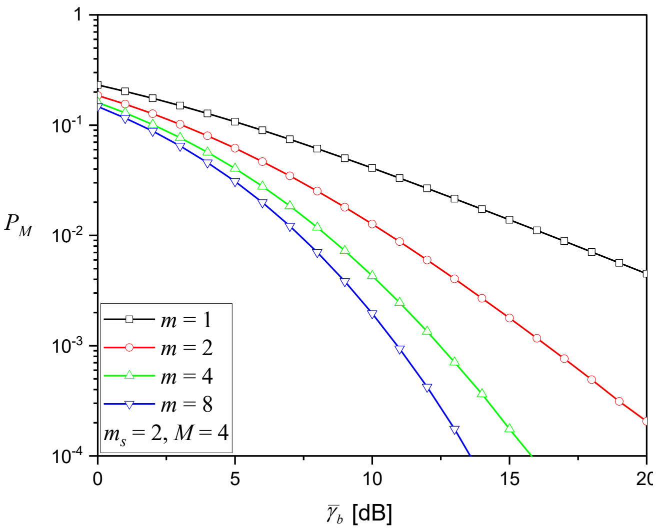

Fig. 4. SER perfomrance for different values of multipath fading severities.

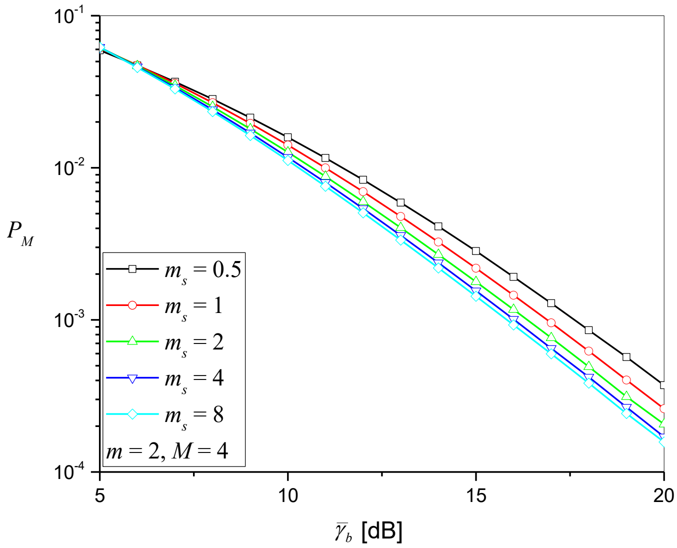

Fig. 5. SER perfomrance for different values of shadowing spread.

V. CONCLUSION

In this paper, we have considered the coherent detection of the MPSK signal transmitted over the channel with thermal noise and shadowed multipath fading. The channel model is characteristic for D2D communications. The emphasis has been placed on determining the required number of terms in series summation when estimating the PDF of the phase of the composite signal. It has been shown that the given accuracy in the summation is certainly achieved with about 750 members of the series, which is performed very quickly on today's computers. We have also given some numerical results that illustrate the influence of channel parameters on the symbol error probability.

Further research will be directed towards the application of differential phase modulation in combination with error correction codes.

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