Method for of objective measurement of acoustic scattering in closed spaces using finite differences in time domain

Ivaylo Stoychev Hristev, Snejana Georgieva Pleshkova

***Abstract*— This article describes the implementation of a MATLAB developed calculation algorithm used for sound scattering investigation. The investigation has been conducted by using standard staggered grid leapfrog Finite Difference in the Time Domain (FDTD) numerical scheme in two dimensions. The shape under investigation is single plane Schroeder sound diffuser. The goal of this article is to apply the Finite Differences in the Time Domain numerical method for objective measurement of sound scattering in enclosed spaces and thus aiding the design and optimisation of room acoustics in such spaces.**

***Index Terms*— FDTD; sound scattering; sound diffusion; Schroeder diffuser; simulation.**

I. Introduction

Recently a discussion is ongoing regarding methods for improvement of diffusiveness of reverberation chambers in order to increase repeatability of absorption materials measurements [1]; another direction of the same discussion is regarding the role of the scattering properties of surfaces in sound field prediction in enclosed spaces [2]. The vast variety of architectural shapes also requires a practical method for sound scattering investigation of arbitrary shapes. Measurement of the sound scattering properties of each surface requires deliberate rooms and equipment, as described by [3], which is tedious and expensive especially during the design process. With the advent of cheap computational power, the numerical methods for simulation of surface's properties is to become inevitable part of the design process.

This article describes the implementation of a MATLAB developed calculation algorithm and the calculation of sound scattering s coefficient; calculations have been performed following the requirements of the aforementioned standards. The results have been compared against data of commercial Quadratic Residue Diffuser (QRD) as well as calculations by the Boundary element Method (BEM).

II.2. Theoretical Basis of Finite Difference in the Time Domain numerical scheme used in objective measures of acoustic scattering

A. Finite Differences in the Time Domain method

A thorough description of the FTDT method has been presented in [4]; here just a brief introduction of the method is presented. The basis of this method are the two fundamental expressions in acoustics – Euler equation of motion and Continuity equation both of which can be found in [1]. Using mathematical line segment representation of time and spatial derivatives and applying it to the aforementioned equations, gives the foundation of the method. Next a grid for calculating the pressure and velocity must be defined, thus forming a discretized domain space in two dimensions. The time is also discretized with fixed step Δt. Because of the interdependence of pressure and velocity a staggered grid must be used. In this arrangement the discretisation in space is achieved by calculating both pressure and velocity in fixed positions, each spaced one step Δx for the x direction and Δy for the y direction apart. Pressure and velocity are spaced ½ spatial step apart and are calculated on each time iteration.

In the two dimensional case, which is of concern in this article, we need to define three grids – one for the pressure derivative, one for x and one for y direction of the velocity vector. Using a fixed discrete time interval Δt the resulting grid update equations are as follows, [5]:

$$u_x^{n+\frac{1}{2}}\left(i+\frac{1}{2},j\right)=u_x^{n-\frac{1}{2}}\left(i+\frac{1}{2},j\right)-\frac{\Delta t}{\rho}\left(\frac{p^n(i+1,j)-p^n(i,j)}{\Delta x}\right); \quad (1)$$

$$u_y^{n+\frac{1}{2}}\left(i+\frac{1}{2},j\right)=u_y^{n-\frac{1}{2}}\left(i+\frac{1}{2},j\right)-\frac{\Delta t}{\rho}\left(\frac{p^n(i+1,j)-p^n(i,j)}{\Delta y}\right); \quad (2)$$

$$p_{(i,j)}^n=p_{(i,j)}^{n-1}-\frac{1}{k}\Delta t\left(\frac{u_x^{n-\frac{1}{2}}\left(i+\frac{1}{2},j\right)-u_x^{n-\frac{1}{2}}\left(i-\frac{1}{2},j\right)}{\Delta x}+\right.$$
$$\left.+\frac{u_y^{n-\frac{1}{2}}\left(i,j+\frac{1}{2}\right)-u_y^{n-\frac{1}{2}}\left(i,j-\frac{1}{2}\right)}{\Delta y}\right). \quad (3)$$

In equations 1 to 3 n denotes the time step and i and j are spatial indexes. The notation $n+½$ is deliberate to the velocity calculation and corresponds to the staggered spatial grid. Here p denotes sound pressure, u denotes particle velocity vector, ρ is the propagation media density, and $k=\rho_0 c^2$ is the compressibility of the medium.

In order to emulate free field conditions either very big discretisation domain should be used, or a proper absorbing boundary should be implemented. In this article a Perfectly Matched Layer (PML) has been used for wave truncation.

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This boundary condition forms a medium with impedance equal to the homogeneous lossless sound propagation media implemented in the discretized domain, but with significant absorption properties which have been initially described by Berenger in [6].

B. Simulated diffuser shape

A well-known and widely investigated shape has been chosen for the purposes of calculation validation – the QRD or Schroeder diffuser shape. The motivation behind this choice is that this is essentially a two dimensional shape, i.e. scatters the sound in a hemicylinder way which can be accurately modelled using a two dimensional FDTD scheme. Figure 1 shows side cut of the QRD as well as a photo of the commercially available product. Sample dimensions are the same as a commercially available product, and are shown in figure 1 and 2. QRD diffusers are so called phase grating sound diffusers, which consists of series of wells with different depth. The width of the wells determines the frequency range, in which the device is effective, while the modulation of the well depth determines the sound scattering properties of the device.

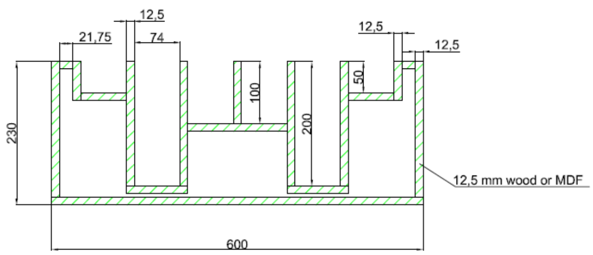

Fig. 1. Phase grating (Schroeder) diffuser section cut

Fig. 2 Photo of commercially available product

A. *MATLAB implementation*

Using equations (1) to (3), an iterative MATLAB algorithm was constructed. On each iteration the pressure and velocity values are calculated and the results kept in two-dimensional variable. The shape of the reflecting shape is defined by a separate "mask" variable, which is multiplied each iteration with the velocity matrix. This method is easy and fast calculation wise and provides zero velocity condition at the boundary of the reflective surface; setting a point with zero velocity generates perfect reflection.

The relevant to acoustics scattering phenomena objective parameter is sound scattering coefficient s [3], which is part of International Standard. Measuring sound scattering requires reverberation field conditions, which would yield enormous calculation time and burden to simulate in a discretized domain. Instead an alternative approach has been taken by using the so called directivity correlation method for calculating scattering. It is thoroughly described in [7] and its main advantage is the usage of anechoic derived data for the sound pressure. In brief it states that by calculating the reflected pressure in an arc of receiver positions, the scattering can be calculated by the following formula

$$s = 1 - \frac{\left| \sum_{i=1}^{n} p_1(v_i) \cdot p_0^*(v_i) \right|^2}{\sum_{i=1}^{n} \left| p_1(v_i) \right|^2 \cdot \sum_{i=1}^{n} \left| p_0(v_i) \right|^2} \tag{4}$$

Here p_1 is the scattered sound field from the surface under consideration, p_0 is the scattered field from a reference smooth plane surface, the asterisk denotes complex conjugate and v_i is the angle of spacing of receivers. Essentially this method takes the correlation between the scattered pressure from surface under investigation and the reflected pressure from reference plane surface. The reference flat surface is a surface having the same external dimensions as the sample, but contrary to the sample is perfectly flat and rigid.

In order to calculate the random incidence value for the coefficient, multiple source positions needs to be accounted. From the data of the various incidence angles, spaced 10 degree apart, the random incidence value is then calculated by the used of Paris formula [8]; grazing incidence point is excluded since it does not contain usable data. The receiver points are the points of virtual microphones, in which the data is being logged. A receiver arc is formed by logging the data in a semicircle array of points spaced 5 degree apart.

Taking into account the considerations for far field assumption, described again in [9], the discretisation domain becomes large if implemented domain is one for all simulations. The large amount of calculation points yields large multiplication matrixes and long calculation time. Figure 3 shows the implementation of the calculation domain with discrete points optimisation. For source positions 5 to 7 a separate larger discretized domain is used. The last two incidence angles do not fit in the larger discretised area and require separate domain, which is wider, but contains less discrete points. The same figure shows also the source and receiver positions, as well as PML strip around the domain.

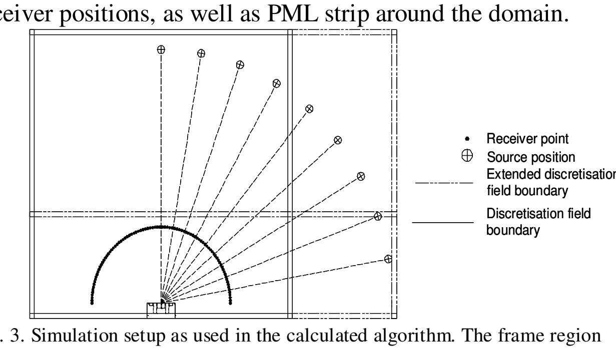

Fig. 3. Simulation setup as used in the calculated algorithm. The frame region around the discretization field denotes PML. Varying the source position is in 10 degree.

II. SIMULATION RESULTS

Simulations have been performed in a discrete domain with 570 discretisation points in each direction. Sound diffuser sample has been located on the bottom end of discrete domain. A sound wave is emitted from a point source which location is varied on a semi-arc with angle increment of 10 degree. Pressure is recorded each time iteration in 180 points on a semicircle with radius corresponding to 1,5 metres. In order to ensure sufficient frequency resolution down to 100 Hz third-octave band, 4096 time step iterations has been performed for each source position; this results in 19.7 Hz frequency resolution, according to the discretization theorem. Summary of all simulation parameters is given in Table 1. For each source angle calculation the pressure has been recorded without reflective surface present, with a diffuser and with a reference plane surface. Later the incident sound wave has been subtracted from the reflected and thus acquiring "clean" response with no artefacts from the incident wave.

Discretisation domain (points)	570x570
Pressure logging points	180
Receivers radius	1,5m.
Time iterations	4096
Frequency resolution	19,7 Hz
Low frequency limit	80 Hz

Table 1 - summary of simlation paramters

Figure 4 shows the angle dependent scattering coefficient, calculated for angles from 0 to 80 degree; also shown with bold line on the figure is the calculated random incidence coefficient. With the increase of incidence angle, the coefficient increases, but the random incidence value is dominated by the angles around 40 to 50 degree; the integration simply adds more weight to these values. Therefore the assumption valid for the absorption coefficient that the value obtained at 45 degree is a good estimate of the random incidence coefficient, is also true for the case of scattering and diffusion coefficients.

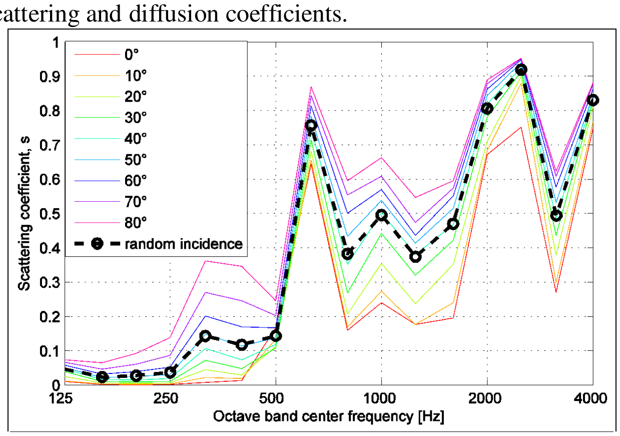

Fig. 4. Calculated angle dependant and random incidence scattering coefficient for QRD sample

Figure 5 compares the random incidence scattering coefficient, calculated by BEM, FDTD, and measurement values from commercial product of manufacturer RPG™. The implemented in [10] BEM calculation algorithm is overestimating the values of diffusion coefficient and this seems like a systematic scaling error. There is lack of frequency points at the high frequency region, above 2 kHz, so the fine dips in the region cannot be accounted for. Since the BEM method is part of another study, the details around which are not completely clear, only the difference between FDTD and commercial sample will be considered relevant. The FDTD method is underestimating the value of scattering coefficient. The difference between FDTD and measured data has an explanation related to the measurement method implemented in practice.

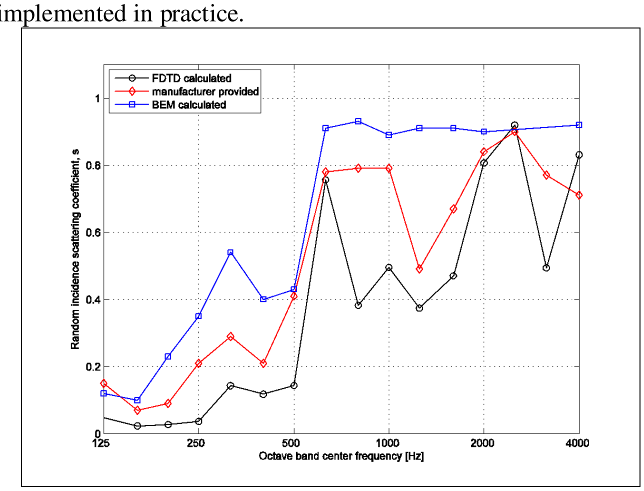

Fig. 5. Random incidence scattering coefficient calculated for QRD sample by FDTD and BEM methods in comparison with commercial product measured data, as provided by manufacturer datasheet

The data, presented on the graphs, is useful in practical terms. The random incidence scattering coefficient is widely used in geometric room acoustic simulation softwares.

III. CONCLUSION

Using classic staggered grid with leapfrog velocity and pressure calculation, it was possible to calculate scattering coefficient for QRD shaped sample. When simulation is considering the sample as totally reflective, there is reasonable agreement between simulated and measured data. The constructed algorithm can be used for calculation of objective sound scattering measures of arbitrary shapes, aiding architectural design of halls and premises. It can also be used for reflector shape optimisation, which is essential when interior of a hall is considered. Scattering properties of wall, ceiling and decorative constructions are crucial for minimizing the error in acoustic simulations of enclosed spaces, and developing of the method is one step forward in this direction.

The low frequency behaviour of an arbitrary shaped room can also be investigated by the use of FDTD scheme. Its advantage of full range simulation with a single calculation can aid the current most widespread design aid – the raytracing algorithm, thus filling the gap in the low frequency region, where raytracing is most vulnerable.

Further development of the method includes investigating sound diffusion coefficient and adding impedance data to the reflective surface. The final aim is to provide useful and accurate tool for sound wave calculation in real world spaces, such as recording studios, concert halls and auditoria.

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