Modeling of the Influence of Corona on the Transmission Lines during Lightning Discharge

Milan Ignjatović, Jovan Cvetić, Nikola Mijajlović, Dragan Pavlović

***Abstract*—The corona discharge in the cylindrical geometry is numerically simulated for the negative lightning voltage impulse. The simulation is performed by solving the drift-diffusion-reaction equations for the electrons, the positive and the negative ions. The results for the concentration of charge particles and the intensity of the electric field are used to determine the corona current which is in turn necessary for the calculation of the transient overvoltages on the transmission lines.**

***Index Terms*—Corona, lightning overvoltage, transmission lines, QV curve**

I. Introduction

Corona is the unwanted effect caused by overvoltages on the transmission lines. There are two significant types of overvoltages in power systems: lightning overvoltage produced by lightning flashes and switching overvoltage produced by switching breakers or disconnecting switches [1]. The level of the transient overvoltage determines the insulation design. Therefore, the accurate calculations of the transient overvoltages are necessary for the designing of the insulation coordination in order to minimize the cost of the construction.

It is important to include the effect of the corona in the calculation for the propagation of voltage or current pulses along the transmission lines. Corona discharge causes the attenuation and the distortion of the pulse. Amplitude of the pulse will be clamped down to the corona threshold at the front of the pulse and the portion of the pulse whose amplitude is larger than the corona threshold will propagate with the speed less than the speed of light [2].

The effects of the corona discharge are nonlinear. Without corona, the dependence of generated charge from the applied voltage is linear and their ratio is a capacitance of the transmission line. If the voltage is high enough, the corona will be formed around the conductor and the charge will start to leak in the surrounding space which makes the charge-voltage function nonlinear. This function is represented by the charge-voltage (QV) curve which can be used to define the dynamical capacitance. The QV curve can be determined experimentally by measuring the charge during the discharge in the cylindrical geometry [3]. Also, different engineering models for the corona are proposed which can be used to evaluate the QV curves [4].

The aim of this paper is to propose the method to take into account the effect of the corona generated by negative lightning overvoltages. The idea is to simulate the corona discharge in the cylindrical geometry under the negative lightning voltage impulse by using the drift-diffusion-reaction equations. Then the QV curve can be calculated and used to determine the dynamical capacitance which figures in the transmission line equations.

The overvoltages caused by lightning can be induced by direct or nearby strokes. The time evolution of lightning effects is described by lightning impulse which is characterized by a small rise time reaching its maximum in a few microseconds and decaying in a few tens of microseconds. The standard lightning impulse 1.2/50 μs can be expressed in analytical form by the sum of two Heidler's functions which are used for the reproduction of the lightning current wave-shape [5].

II. Theory

A. The corona discharge

The time evolution of the space densities of the charged particles during the discharge is usually described by drift-diffusion-reaction equations [6] given by

$$\frac{\partial n_e}{\partial t}+\frac{1}{r}\frac{\partial\left(r\Gamma_e\right)}{\partial r}=n_e\alpha\left|W_e\right|-n_e\eta\left|W_e\right|-n_e n_p\beta\,, \tag{1}$$

$$\frac{\partial n_p}{\partial t}+\frac{1}{r}\frac{\partial\left(r\Gamma_p\right)}{\partial r}=n_e\alpha\left|W_e\right|-n_e n_p\beta-n_n n_p\beta\,, \tag{2}$$

$$\frac{\partial n_n}{\partial t}+\frac{1}{r}\frac{\partial\left(r\Gamma_n\right)}{\partial r}=n_e\eta\left|W_e\right|-n_n n_p\beta\,, \tag{3}$$

where n_e, n_p and n_n are the densities of the electrons, the positive and the negative ions, respectively. On the right hand side of these equations the terms representing the gain and the loss of the particles due to ionization, attachment and

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recombination described by the coefficients α, η, β, respectively, are given. Further W_e, W_p and W_n are the velocities of the electron, the positive and the negative ion drifts, respectively. The fluxes of charged particles are given by

$$\Gamma_e = W_e n_e - D\frac{\partial n_e}{\partial r}, \tag{4}$$

$$\Gamma_p = W_p n_p, \tag{5}$$

$$\Gamma_n = W_n n_n, \tag{6}$$

where Γ_e is the flux of the electrons, Γ_p and Γ_n are the fluxes of the positive and the negative ions, respectively, D is the diffusion coefficient for the electrons. The values of the transport and the reaction coefficients depend on the value of the electric field intensity at each point. Therefore the continuity equations (1), (2) and (3) are coupled with the Poisson equation using the potential Φ

$$\nabla^2\Phi = -\frac{e\left(n_p - n_e - n_n\right)}{\varepsilon_0}. \tag{7}$$

Another effect that must be taken into account to the negative corona discharge is the emission of the electrons due to the positive ion impact with the cathode. The flux of the electrons on the surface of the cathode is given by

$$\Gamma_e = \gamma\,\Gamma_p, \tag{8}$$

which is used as boundary condition in the numerical simulations. Here, γ is the ion-secondary emission coefficient. Its value does not depend much on the type of the cathode material, therefore the usual value $\gamma = 0.01$ is adopted.

B. The total charge density

If the concentrations of the charge particles and the electric field intensity are calculated, the total line charge density is

$$Q(t) = \int_0^t I_p(t')dt', \tag{9}$$

where I_p is the current per cylinder length, obtained from Sato-Morrow [7] formula

$$I_p = \frac{e}{V_p}\int \vec{\Gamma}\cdot\vec{E}_L dS + \frac{\varepsilon_0}{V_p}\int\frac{\partial\vec{E}_L}{\partial t}\cdot\vec{E}_L dS. \tag{10}$$

Here $\vec{\Gamma} = \vec{\Gamma}_p - \vec{\Gamma}_e - \vec{\Gamma}_n$, $dS = r dr d\varphi$ is the cross section of the elementary surface in the cylindrical coordinate system, V_p is the voltage impulse and $\vec{E}_L$ is the Laplacian electric field representing the electric field that would exist between the electrodes if there was no generated space charge. For the coaxial wire-cylinder configuration one obtains

$$E_L = \frac{V_p}{r\ln\frac{R_2}{R_1}}, \tag{11}$$

where R_1 and R_2 are the wire and the cylinder radii, respectively. The expression for I_p becomes

$$I_p = \frac{2\pi e}{\log\frac{R_2}{R_1}}\int_{R_1}^{R_2}\Gamma dr + \frac{2\pi\varepsilon_0}{\log\frac{R_2}{R_1}}\frac{\partial V_p}{\partial t}. \tag{12}$$

The first term in (12) is the current component due to charge motion in the ionized gas. The second term is the capacitive current component which flows through the cylindrical capacitor due to the time varying voltage.

The total line charge density obtained by (9) is the total line charge that is flown through the external circuit since the time onset of the voltage impulse. It can be also calculated in a different way. It can be considered as the sum of two components

$$Q = Q_\rho + Q_C, \tag{13}$$

where Q_ρ is the corona charge in the space between the electrodes and Q_C is the charge residing on the central wire both per the cylinder length. The Q_ρ component can be calculated when the space charge $\rho = e\left(n_p - n_e - n_n\right)$ over the cross section is integrated. The value of the Q_C component enables that the integral of the total electric field is equal to the applied voltage at any time.

The total electric field at the outer cylinder can be calculated now as

$$E\left(r = R_2\right) = \frac{Q_\rho + Q_C}{2\pi\varepsilon_0 R_2}. \tag{14}$$

Two charge components can be now calculated as

$$Q_\rho = 2\pi\int_{R_1}^{R_2}\rho r dr, \quad Q_C = 2\pi\varepsilon_0 R_2 E\left(R_2\right) - Q_\rho. \tag{15}$$

C. The transmission line equations

The voltage induced on an overhead wire above the ground can be calculated using the transmission line equations [2]

$$\frac{\partial V(t,x)}{\partial x}+L\frac{\partial I(t,x)}{\partial t}=0, \tag{16}$$

$$\frac{\partial I(t,x)}{\partial x}+C\frac{\partial V(t,x)}{\partial t}=-I_{cor}(t,x), \tag{17}$$

where V and I are the voltage and the current along the line, L and C are the line inductance and the line capacitance, respectively, and I_{cor} is the corona current line density. In the cylindrical geometry one can obtain the time dependence for the corona current $I_{cor}(t)$ as a time derivative of the corona line charge density q_{cor} which is the difference between the total charge calculated by (13) and the charge C_0V_p in the cylindrical capacitor without the corona

$$I_{cor}=\frac{\partial q_{cor}}{\partial t}=\frac{\partial}{\partial t}\left(Q-C_0V_p\right), \tag{18}$$

where $C_0=2\pi\varepsilon_0/\ln\left(R_2/R_1\right)$ is the line capacitance.

In order to determine the corona charge for the wire of radius R_1, we need to assume the value for radius of the outer cylinder R_2 for the calculations. The value of the radius R_2 is chosen so that the electric field does not reach the critical value for the ionization at that distance. Further, the amplitude of the voltage impulse should be assumed so that the electric field around the central wire corresponds to the expected field around the overhead wire.

For the sake of the simplicity, it is assumed that the corona current at each line segment is identical in magnitude and shape, so

$$I_{cor}\left(t,z\right)=I_{cor}\left(t-z/c\right), \tag{19}$$

where c is the speed of light.

III. Results and Discussion

We have simulated the discharge between the wire of radius $R_1=5$ mm and the cylinder of radius $R_2=40$ cm. The outer cylinder is grounded and the standard negative lightning voltage impulse whose amplitude is 267 kV is applied to the central wire. This configuration have been used in the experiments performed by Cooray [4]. The experimental measurements of the QV curves for different amplitudes of the positive and the negative lightning voltage impulses are presented in that study enabling the comparison of our results to the measured values. Cooray [4] also proposed a simple engineering model for the QV predictions that is widely used in the literature.

The time evolution of the total charge density is presented in Fig. 1 with the components Q_C and Q_ρ. Also, the charge of the cylindrical capacitor without corona $Q_0=C_0V_p$ is shown. In the beginning of the discharge the total charge density is equal to Q_0 until the corona starts to disturb the Laplacian electric field and it contains only the Q_C component. When the positive ions start to leave the inter-electrode space through the cathode approximately at $0.8\ \mu$s, the Q_C component decreases abruptly and the Q_ρ component becomes dominant until the end of the simulation.

When the charge reaches its maximum value at $8\ \mu$s, the Q_C component decreases to zero value and later becomes positive. This is due to the decreasing value of the negative applied voltage.

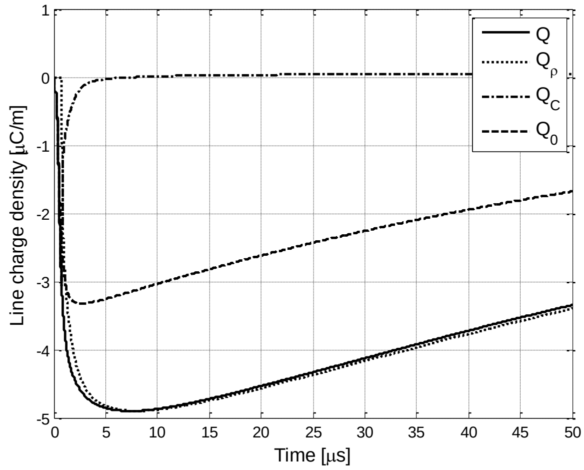

Fig. 1. The line charge density and its components

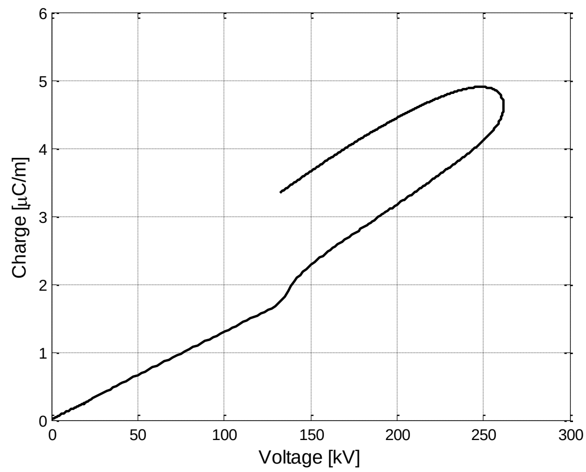

Fig. 2. The calculated QV curve

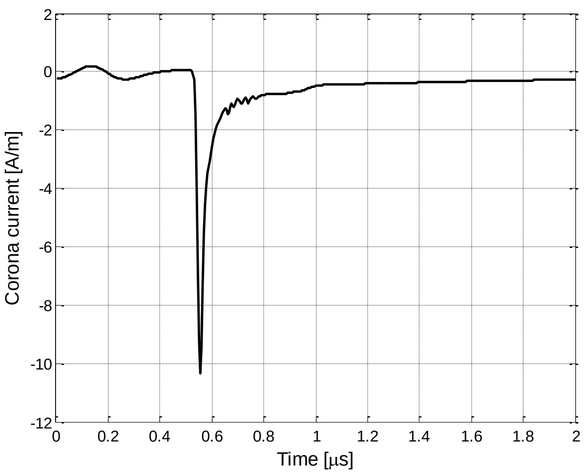

Fig. 3. Corona current density

The calculated QV curve (Fig. 2.) is in a good agreement with the experimental measurements presented in [4] and it has maximum value of about $5\,\mu\mathrm{C}$. In the rise portion of the voltage impulse the relation between the Q and V is linear. The linear dependence stops when the voltage reaches the value of 130 kV. It is the moment of the corona inception. The curve has hysteresis because the charge accumulated in the inter-electrode space during the rise portion of the voltage remains in the system or decreases slower than the voltage.

Finally, the calculated corona current density is shown in Fig. 3. In the beginning it has a negligible value until the corona inception starts. Then a very narrow pulse occurs and its value decreases to zero.

IV. Conclusion

The determining of the QV curve is necessary in order to include the effects of the corona in the calculation of transient lightning overvoltages. Various engineering models are proposed for the evaluation of the QV curve based on very simplified physical assumptions of the discharge process. In this paper, we have simulated the corona discharge by solving the drift-diffusion equations directly without any constraints. The results show a good agreement with the available experimental data. They can be used for further studying of the corona discharge and its effect on the voltage pulse propagation along the transmission lines.

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