

A research on the influence of non-ideal analog-to-digital converter on the measurement error of frequency

Andrey Serov, Alexander Shatokhin, Ivan Konchalovsky

Abstract—In this paper, we consider a method for measuring frequency, based on the determination of voltage transitions through zero (zero crossing technique). A brief description of this measurement method is given, as well as its modification, based on the linearization of the signal near zero. The use of this modification significantly improves the accuracy of frequency measurement. The analysis of the influence of the non-identity of the applied analog-to-digital converter (ADC) on the frequency measurements error was performed. The influence of additive, multiplicative error components, errors caused by the nonlinearity of the ADC conversion function and the error components caused by the quantization error is considered. It was shown that additive and multiplicative error components do not significantly influence the error of frequency measurements. Analytical relations that allow one to estimate the measurement error of the frequency from the ADC quantization error and the nonlinearity of the ADC conversion function were obtained. The reliability of the obtained results was confirmed by applying mathematical modeling (with the software Simulink 8).

Index Terms—Measurement Error, Frequency Deviation, Analog-to-Digital Converter, Integral Nonlinearity, Zero Crossing Technique.

I. INTRODUCTION

NOWADAYS, there are large numbers of digital methods for measuring frequency [1]–[18] of the power system. Note that in real power networks the voltage is not sinusoidal and has the following features: there is a finite frequency deviation of the signals from the nominal value; the spectral composition of the network voltages and currents contains several harmonics; there may be a flicker or noise.

The features considered make it difficult to accurately measure the frequency. All digital methods of frequency measurement can be divided into three categories: based on the representation of signals in the time, based on the representation of signals in the frequency and combined methods of representation.

Andrey Serov is with the Measurement Technique Department, National Research University “Moscow Power Engineering Institute”, 14 Krasnokazarmennaya st., 111250 Moscow, Russia (e-mail: serov.an.iit@yandex.ru).

Alexander Shatokhin is with the Measurement Technique Department, National Research University “Moscow Power Engineering Institute”, 14 Krasnokazarmennaya st., 111250 Moscow, Russia (e-mail: sh1@yandex.ru).

Ivan Konchalovsky is with the Measurement Technique Department, National Research University “Moscow Power Engineering Institute”, 14 Krasnokazarmennaya st., 111250 Moscow, Russia (e-mail: Hummel-80@yandex.ru).

The frequency of the network can be found by performing spectral analysis and determining the frequency of the largest component of the amplitude spectrum. This principle underlies the group of methods of representation in the frequency domain. Since the resolution of the amplitude spectrum with respect to frequency is finite, then using these methods a discretization error arises. To reduce this error, methods based on the approximation of the discrete spectrum by means of various functions have been developed.

The methods of "adaptive filtering" [1], [2] and "from the derivative of the input signal" [2]–[4] are intended for measuring the frequency of sinusoidal waveforms. The presence of harmonics leads to the appearance of large additional measurement errors. The method based on the determination of voltage transitions through zero [2], [5], [6] is based on the determination of the moments of time at which the zero crossing of the input voltage occurs. Many modifications of this method are based on the approximation of the signal (voltage) near its zero crossing to increase the accuracy of finding the moment of intersection. The main drawback of the method is associated with the occurrence of an additional error in the presence of harmonics. However, this drawback can be eliminated by using a bandpass or a low pass filter.

The group of combined methods is based on the implementation of the sliding discrete Fourier transform [2], [7]–[9] and the subsequent application of one of the methods of representing the signal in the time domain. Since the sliding discrete Fourier transform has a frequency response corresponding to a bandpass filter, the methods under consideration reduce the influence of both the flicker and the harmonics. The main drawback of the methods under consideration lies in their relatively high computational complexity.

Among methods for measuring frequency in the time domain, the most widely used method is based on the determination of the signal transitions (in our case, the voltage) through zero [2], [5], [6] – zero-crossing technique. There are many publications in which the successful application of this method for measuring the frequency of voltage of today power networks is considered. However, the question of the influence of non-ideal ADC (mostly the ADC quantization error and the nonlinearity of its conversion function) on the error in measuring the frequency when applying this method has not been thoroughly studied.

Then the moment of zero crossing can be found by solving the following equation:

$$a_k(n_{cz} + \eta_{cz})^k + a_{k-1}(n_{cz} + \eta_{cz})^{k-1} + \dots + a_0 = 0, \quad (6)$$

where n_{cz} – sample number (integer value) closest to the intersection by the input signal to zero, found per the equation (6); η_{cz} – correction of the reference number corresponding to the moment when zero intersects the input signal (n_{cz}).

If there are several roots in the solution of equation (6), the root corresponding to the intersection of zero by the input signal must satisfy the condition $n_{cz} \leq (n_{cz} + \eta_{cz}) \leq n_{cz} + 1$, where $(n_{cz} + \eta_{cz})$ – is the root of equation (6). The value of the time of zero crossing for the obtained roots of equation (6) is defined as:

$$t_{cz} = (n_{cz} + \eta_{cz})T_S = n_{cz}T_S + \Delta t_{cz}. \quad (7)$$

Since the sinusoidal signal is close to linear upon crossing zero, it is justified to use approximating polynomials of the first order. The fraction of the sampling period between the voltage transition through zero and the first sample after this is determined by the expression [5]:

$$t_{cz} \cong T_S n_{cz} + T_S \eta = \frac{T_S n_{cz} u[n_{cz} + 1] - T_S (n_{cz} + 1) u[n_{cz}]}{u[n_{cz} + 1] - u[n_{cz}]}. \quad (8)$$

Then the correction values η_0 , η_m for (2) and (3) can be found using the expression for the first transition for the i -th measurement interval:

$$\eta_{cz,0} = \frac{u[n_{cz,0}]}{u[n_{cz,0}] - u[n_{cz,0} - 1]}, \quad (9)$$

and for the first transition for $(i+1)$ -th measurement interval:

$$\eta_{cz,m} = \frac{u[n_{cz,m}]}{u[n_{cz,m}] - u[n_{cz,m} - 1]}, \quad (10)$$

where $u[n_{cz,0}]$ – first positive voltage indication on the i -th measurement interval; $u[n_{cz,0} - 1]$ – last negative voltage indication on the $(i - 1)$ -th measurement interval; $u[n_{cz,m}]$ – first positive voltage sample on the $(i + 1)$ -th measurement interval; $u[n_{cz,m} - 1]$ – last negative voltage sample on the i -th measurement interval.

Then the correction value η :

$$\eta = \frac{u[n_{cz,0}]}{u[n_{cz,0}] - u[n_{cz,0} - 1]} - \frac{u[n_{cz,m}]}{u[n_{cz,m}] - u[n_{cz,m} - 1]}. \quad (11)$$

To simplify further transformations, we take the following notation:

$$\begin{cases} X_1 = u[n_{cz,0} - 1], & X_2 = u[n_{cz,0}], \\ X_3 = u[n_{cz,m} - 1], & X_4 = u[n_{cz,m}]. \end{cases} \quad (12)$$

With expressions (9), (10) and (12), the value of the frequency of the input voltage is determined by (see expression (2)):

$$\begin{cases} f = \frac{mf_S}{N + \eta}, \\ \eta = \frac{X_2}{X_2 - X_1} - \frac{X_4}{X_4 - X_3}. \end{cases} \quad (13)$$

There are papers [2], [5], [6] devoted to the zero crossing technique and its modifications. However, there is no analysis of the sources of errors caused by the imperfection of the ADC in these papers. There are also no analytical relations for estimating additional errors caused by the linearization of the voltage near the crossing of zero. The estimates given in these studies were carried out by mathematical modeling. This makes it difficult to analyze and evaluate the measurement error when applying this method.

III. MEASUREMENT ERRORS

The imperfection of the ADC leads to an inaccurate definition of η and, consequently, to the error f_i . We denote by $\Delta\eta$ the absolute error η due to the imperfection of the ADC. Using formula (13), we express the relative error of frequency measurement δ_f via $\Delta\eta$:

$$\delta_f = -\frac{\Delta\eta}{N + \eta} \cong -\frac{\Delta\eta}{N}. \quad (14)$$

The approximate equation (14) can be used in calculations, since usually $\Delta\eta \ll N$. The error of the correction can reach 100 %, when the derivative of the voltage approaches zero when passing through zero. This situation can be caused by voltage harmonics. In the worst-case $X_2 = X_4 = 0$ (zero is considered positive); $X_1 = X_3 = q$ (q – value of the quantum of the ADC). To eliminate this, the ADC samples must be digitally filtered. To do this, it is sufficient to use a low-pass or bandpass filter. Since the derivatives of the harmonics are proportional to their number, the derivatives of the harmonics at the output of the second-order filter will be inversely proportional to their number. It is further assumed that preliminary digital filtering of ADC samples is used, which are then used in frequency measurement. During digital filtering, both IIR and FIR filters can be used to suppress noise and harmonics. It should be considered that the implementation of a FIR filter with a narrow bandwidth requires a much larger filter order than during IIR filter implementation. Express $\Delta\eta$ with ΔX_1 , ΔX_2 , ΔX_3 , ΔX_4 – absolute sampling errors of X_1 , X_2 , X_3 , X_4 :

$$\Delta\eta = \sum_{i=1}^4 \left| \frac{d\eta(X_1, X_2 \dots X_4)}{d(X_i)} \right| \Delta X_i, \quad (15)$$

where $[X_1, X_2, X_3, X_4]$ – are the influencing values; ΔX_i – marginal deviation of the influencing value X_i .

With (13) and (15):

$$\Delta\eta = \frac{X_2 \cdot \Delta X_1 - X_1 \cdot \Delta X_2}{(X_2 - X_1)^2} - \frac{X_4 \cdot \Delta X_3 - X_3 \cdot \Delta X_4}{(X_4 - X_3)^2}. \quad (16)$$

Then the measurement error of the frequency:

$$\delta_f \cong \frac{X_4 \cdot \Delta X_3 - X_3 \cdot \Delta X_4}{N(X_4 - X_3)^2} - \frac{X_2 \cdot \Delta X_1 - X_1 \cdot \Delta X_2}{N(X_2 - X_1)^2}. \quad (17)$$

Relations between (16) and (17) do not consider the nature of the deviations of the influencing quantities and their relationship among themselves. That is why, the direct application of this approach will lead to an overestimation of the error. Considering the sources of error in measuring instantaneous voltage values and their effect on the error of frequency measurement. The following components are caused by the non-ideal ADC:

- the additive component caused by the zero offset of the ADC (Δ_{ADD});

- the multiplicative component caused by the deviation of the transfer coefficient of the ADC converter and the deviation of the ADC reference voltage source from the nominal value (the total error (δ_{MUL}));

- linearity caused by the nonlinearity of the ADC conversion function (Δ_{LIN});

- the quantization error of the ADC.

For the additive component, the error in measuring instantaneous voltage values:

$$\Delta X_1 = \Delta X_2 = \Delta X_3 = \Delta X_4 = \Delta_{ADD}. \quad (18)$$

The values of the voltage readings for which the corrections of η_0 are calculated, along η_m modulus are close to each other. Since these samples are located near one level (zero), we can write:

$$|X_1| = |X_2| = |X_3| = |X_4|. \quad (19)$$

From expression (17), (18) and (19), the error in measuring the frequency from the additive component of the ADC error is zero.

For the multiplicative component of the error, considering condition (19):

$$\Delta X_1 / X_1 = \Delta X_2 / X_2 = \Delta X_3 / X_3 = \Delta X_4 / X_4 = \delta_{MUL}. \quad (20)$$

Thus, the error in measuring the frequency is affected only

by the quantization errors and the linearity errors of the ADC. The linearity error can vary strongly (up to 100% in the case of pipeline ADC and successive approximation ADC), even for neighboring values of the ADC conversion function.

The error in measuring the frequency due to the aperture delay and the aperture bounce of the ADC can be neglected, as shown by the results of mathematical modeling.

Let us proceed to estimate $\Delta\eta_{max}$ – maximum value of the module $\Delta\eta$. That is why we formulate a number of propositions concerning $\Delta\eta$, on which the assessment will be based $\Delta\eta_{max}$. First, all absolute errors in ADC samples are pairwise independent of each other and can be positive or negative. Therefore, in the worst case, they are summed up. The maximum value of these errors (ΔX_x) in the least significant bit (LSB) of the ADC are determined by the quantization error ($1/2$) and INL – the integral nonlinearity of the ADC:

$$\Delta X_x = 0.5 + INL. \quad (21)$$

Secondly, the increments in the output signal of the ADC are the same and are determined by the derivative of the sinusoidal voltage at the zero point, q is the LSB of the ADC and the sampling frequency f_s :

$$X_2 - X_1 \cong X_4 - X_3 \cong \frac{2\pi f_1 U_m}{q \cdot f_s}, \quad (22)$$

where U_m – amplitude of sinusoidal voltage.

A LSB, in units of voltage, is defined:

$$q = \frac{2U_{ref}}{2^{N_{ADC}-1}} = \frac{2U_m}{2^{N_{ADC}-1}}, \quad (23)$$

where N_{ADC} – ADC resolution; coefficient 2 in the numerator considers the requirements of normative documentation: the upper limit of the voltage measurement range equals to twice the nominal value of the voltage.

With (16), (21) – (23) equation for $\Delta\eta_{max}$:

$$\Delta\eta_{max} = \frac{(1 + 2INL)f_s}{\pi f_1 2^{N_{ADC}-1}}. \quad (24)$$

Then (14) и (24), $\delta_{f,max}$ – maximum relative error of the frequency measurement, caused by the imperfection of the ADC:

$$\delta_{f,max} = \frac{\Delta\eta_{max}}{N} = \frac{(1 + 2INL)f_s}{\pi N f_1 2^{N_{ADC}-1}} \quad (25)$$

With (2), approximately expressing N :

$$N \cong \frac{m f_s}{f_1}. \quad (26)$$

And substituting its expression in (25). Thus, we finally find:

$$\delta_{f,\max} = \frac{\Delta\eta_{\max}}{N} = \frac{1 + 2INL}{\pi m 2^{N_{ADC}-1}}. \quad (27)$$

IV. SIMULATION RESULTS

Equation (27) shows that $\delta_{f,\max}$ is determined only by the ADC parameters characterizing its nonideality, and the duration of the measurement interval in the periods of the fundamental voltage component. Table 1 presents the calculated values of $\delta_{f,\max}$ for some types of ADC.

TABLE I
ADC PARAMETERS AND FREQUENCY MEASUREMENT ERROR

ADC Parameter	ADC		
	ADS7812PB	ADS7279	ADS8513IB
Resolution	12	14	16
Offset error, max (mV)	6	1.25	6
Full-scale error, max (%)	0.25	-----	0.25
Gain error, max (%)	-----	0.25	-----
INL, max (LSB)	0.5	1.0	2.0
Relative frequency measurement error, δf (%)			
Mathematical modeling (Simulink 8)	$6,3 \cdot 10^{-4}$	$8,1 \cdot 10^{-7}$	$2,6 \cdot 10^{-5}$
Equation (27)	$3,1 \cdot 10^{-3}$	$1,2 \cdot 10^{-3}$	$4,9 \cdot 10^{-4}$

The error is calculated for a number of modern ADC from Texas Instruments (ADS7812PB, ADS7279, ADS8513IB). When performing the calculation, it is assumed that the only source of instrumental error is the non-ideality of the ADC: the quantization error and non-linearity of the conversion function. The parameters of the considered ADC are taken from the technical documentation posted on the manufacturer's website – Texas Instruments (www.ti.com). The results corresponding to the line "Mathematical modeling" were obtained by applying by Simulink 8.

The results are obtained with the following values of the parameters: the sampling frequency – 10240 Hz; measurement time – 0,2 sec (10 nominal periods of input voltage); fundamental frequency is 50 Hz; range of input voltage variation corresponds to the input range of the ADC.

V. CONCLUSION

For zero crossing technique, a modification is considered that includes linear approximation of the voltage near the zero crossing to calculate the correction to improve the accuracy of the frequency measurement. An analytical expression (27) is obtained that allows one to estimate the maximum error of

frequency measurement caused by the imperfection of the ADC: the quantization error and the nonlinearity of the conversion function. The results of mathematical modeling show that the application of the expression (27) obtained allows us to estimate from above the value of the measurement error in the frequency using this method. The expression can be used to select the ADC resolution that provides voltage frequency measurements with a given error.

REFERENCES

- [1] D. Ould Abdeslam, P. Wira, J. Mercklé, D. Flieller. "A neural learning approach for time-varying frequency estimation of distorted harmonic signals in power systems", 1st International ICSC Symposium on Industrial Application of Soft Computing (IASC 2005), 2005.
- [2] B. Boashash. "Estimating and interpreting the instantaneous frequency of a signal. II. Algorithms and applications", Proceedings of the IEEE, vol. 80, issue 4, pp. 540–568, 1992.
- [3] Wu Jiekang "Wide range parameter measurement of sinusoidal signals", Journal of electrical engineering, vol. 57, issue 2, – pp. 111–115, 2006.
- [4] Wu Jie-kang, He Ben-teng. "An algorithm for frequency estimation of signals composed of multiple single-tones", Journal of Zhejiang University Science A, vol. 7, issue 2, pp. 179–184, 2006.
- [5] P.M. Djuric, M.M. Begovic, M. Doroslovacki. "Instantaneous phase tracking in power networks by demodulation", IEEE Transactions on Instrumentation and Measurement, vol. 41, issue 6, pp. 963–967, 1992.
- [6] M.M. Begovic, P.M. Djuric, S. Dunlap, A.G. Phadke. "Frequency tracking in power networks in the presence of harmonics", IEEE Transactions on Power Delivery, vol. 8, issue 2, pp. 480–486, 1993.
- [7] D. Agrez. "Power system frequency estimation in the shortened measurement time", Proceedings of the 18th IEEE Instrumentation and Measurement Technology Conference, vol. 2, pp. 1094–1098, 2001.
- [8] T. Lobos, J. Rezmer. "Real-time determination of power system frequency", IEEE Transactions on Instrumentation and Measurement, vol. 46, issue 4, pp. 877–881, 1997.
- [9] D. Belega, D. Dallet. "Frequency estimation via weighted multipoint interpolated DFT", IET Science, Measurement & Technology, vol.2, issue 1, pp. 1–8, 2008.
- [10] P.J. Moore, J.H. Allmeling, A.T. Johns. "Frequency relaying based on instantaneous frequency measurement power systems", IEEE Transactions on Power Delivery, vol. 11, issue 4, pp. 1737–1742, 1996.
- [11] T.S. Sidhu, M.S. Sachdev. "An iterative technique for fast and accurate measurement of power system frequency", IEEE Transaction on Power Delivery, vol. 13, issue 1, pp. 109–115, 1998.
- [12] T.S. Sidhu. "Accurate measurement of power system frequency using a digital signal processing technique", IEEE Transactions on Instrumentation and Measurement, vol. 48, issue 1, pp. 75–81, 1999.
- [13] S.N. Mikhailin, V. M. Gevorkyan. "The problems of digital processing of signals in the system of automated power quality control and accounting quantity of electricity (ASQAE)", MPEI Vestnik, No. 1, pp. 86-92, 2005.
- [14] A.V. Bunin, V.M. Gevorkyan, Y.A. Kazantsev, S.N. Mikhailin, B.S. Novikov. "Integrated measuring device the automated system of accounting of quantity and quality control of electric energy in high voltage networks", Elektro, No.5, pp. 18-22, 2003.
- [15] M.D. Kusljevic. "A simple recursive algorithm for frequency estimation", IEEE Transactions on Instrumentation and Measurement, vol. 53, issue 2. pp. 335–340, 2004.
- [16] M.D. Kusljevic, J.J. Tomic, L.D. Jovanovic. "Frequency Estimation of Three-Phase Power System Using Weighted-Least Square Algorithm and Adaptive FIR Filtering", IEEE Transactions on Instrumentation and Measurement, vol. 59, issue 2. pp. 322–329, 2010.
- [17] M.D. Kusljevic. "A Simultaneous Estimation of Frequency, Magnitude, and Active and Reactive Power by Using Decoupled Modules", IEEE Transactions on Instrumentation and Measurement, vol. 59, issue 7, pp. 1866–1873, 2010.
- [18] P.D. Poljak, M.D. Kusljevic, J.J. Tomic. "Power Components Estimation According to IEEE Standard 1459–2010 Under Wide-Range Frequency Deviations", IEEE Transactions on Instrumentation and Measurement, vol. 61, issue 3, pp. 636–644, 2012.