THD Factor Measurement Optimization Using Stochastic Method

Velibor Pjevalica, Nebojša Pjevalica, Nenad Petrović and Nikola Teslić

***Abstract* — In this paper, optimization of the stochastic approach for THD factor measurement is shown. There are two THD factor definitions. By one definition, THD factor is the true RMS of all harmonics higher than the first divided with the amplitude of the first harmonic. This is THD fundamental definition. THD root mean square definition is the true RMS of all harmonics higher than the first divided with true RMS of all harmonics, including first. Due to high precision and flexibility of the stochastic method, it is possible to measure THD factor directly, but it is also possible to optimize measurement by measuring first harmonic and true RMS of analog difference between analog signal and its first harmonic.**

***Index Terms* — THD Factor, true RMS, A/D conversion, THD fundamental, THD root mean square**

I. Introduction

There are two THD factor definitions THD that are in use today:

– THD Fundamental

– THD root mean square

THD fundamental is used in situations when it is important to measure quality i.e. "purity" of the measured signal. It means that we want to have information about higher harmonics to fundamental harmonics ratio. In practice, for real signals this ratio is always smaller then one, but theoretically it can be higher then unity as well. Definition [1] of THD_F factor is given with:

$$THD_F = \frac{\sqrt{\sum_{i=2}^{N} V_i^2}}{V_1} \qquad (1)$$

In here V_i, (i=1,2,…,N) are harmonics amplitudes, where i is order of the harmonic. Number N is the highest order of the harmonic analyzed. Definition is [1] THD_R given with:

$$THD_R = \frac{\sqrt{\sum_{i=2}^{N} V_i^2}}{\sqrt{\sum_{i=1}^{N} V_i^2}} \qquad (2)$$

According to THD_R definition, it is impossible for THD_R to be higher than one.

THD_F and THD_R are interconnected with:

$$THD_R = \frac{THD_F}{\sqrt{1+THD_F^2}} \qquad (3)$$

In another words, knowing one THD factor for given signal, we can calculate the other. If distortion of the measurement signal is small, i.e. if presence of higher order harmonics is not significant and most of the signal energy is in fundamental harmonic, then those two THD factors are almost the same.

Today more and more industrial converters in distributive network significantly influence on harmonic structure of both voltage and particularly current signal [2-5]. It is not only industrial converters that changes harmonic structure in distributive network but also consumer electronics devices contributes in THD factor raise [6].

In order to measure THD factor, we need to precisely and accurately measure all harmonic values of the measured signal. There are alternative attempts for THD measurement by using discrete wavelet transform [7]. However, measurement with DFT coefficients gives more reliable result since in that case measurement is done strictly according to definition. In that sense, generalized stochastic processor of orthogonal transformations [8] is good starting point for measurement of THD factor.

This paper structure is following: in section II stochastic processor of orthogonal transformation is presented. In section III theory of operation is given. Section IV shows simulation results. Section V is discussion, and section VI gives conclusion. In section VII there is reference list.

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II. STOCHASTIC PROCESSOR OF ORTHOGONAL TRANSFORMS

Stochastic adding analog-to-digital conversation with two noise generator [8] (abbreviated SAADK 2G) is oversampling measurement method. With this method, we digitally measure averaged value of the integral of two analog input signals product. Let's denote these signals with $y_1(t)$ and $y_2(t)$. Block scheme of SAADK 2G is given in fig 1.

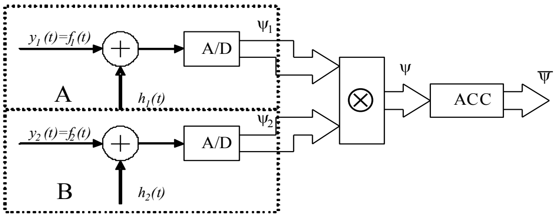

Fig. 1. Block scheme of SAADK 2G converter

As it can be seen in fig 1, it is necessary to add in analog manner (analog addition) noises $h_1(t)$ and $h_2(t)$ to measured signals $y_1(t)$ and $y_2(t)$ respectively. Conditions for operation of SAADK 2G is that noise signals are mutually uncorrelated and that distribution of amplitudes is uniform within the range of $\pm\Delta_i/2$, i = 1, 2. Δ_i are steps of A/D converters 1 and 2 from fig 1 respectively. If these conditions are fulfilled, numerical accumulator ACC from fig 1 contains number $\overline{\psi}$ that is:

$$\overline{\psi} = \frac{1}{T}\int_0^T y_1(t)y_2(t)dt\,. \tag{4}$$

Upper bound for absolute measurement error is:

$$\sigma_s^2 = \frac{1}{T}\frac{\Delta_1^2}{4}\int_0^T y_2^2(t)dt + \frac{1}{T}\frac{\Delta_2^2}{4}\int_0^T y_1^2(t)dt + \frac{\Delta_1^2\Delta_2^2}{16} \tag{5}$$

Block B in figure 1 can be replaced with memory block. This memory block holds dithered sample of basis functions. Such instrument is called stochastic processor of orthogonal transformations [8] and is shown schematically in fig 2.

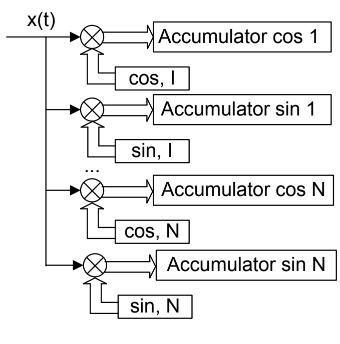

Fig. 2. Block scheme of stochastic processor of orthogonal transformations

In reference [8] it is shown that optimal resolution of the stored samples is exactly 2 bits higher then input analog-to-digital convertor. In that case, waveform of measured signal does not influence the measurement accuracy. Optimization is made on hardware complexity vs accuracy criterion.
If this condition is satisfied, measurement uncertainty is:

$$\sigma_s^2 \approx \frac{1}{N}\frac{\Delta_1^2}{8}R^2 \tag{6}$$

In here, R is input voltage range in A/D converter, N is overall number of samples in the measurement result and Δ_1 is step of analog-to-digital converter.

If we take resolution of stored samples of 12 bits, optimal resolution of analog-to-digital converter is 10 bits. Input voltage range in A/D converter is R=2.5V. Sampling frequency is 1MHz (20,000 samples per period for 50Hz mains). Overall duration of the measurement is 2 s. With given measurement parameters, upper bound limit for absolute measurement error is:

$$\sigma_s = \sqrt{\frac{1}{2\cdot\left(2\cdot 10^4\right)}\frac{\left(\frac{2{,}5}{2^{10-1}-1}\right)^2}{8}2{,}5^2} \tag{7}$$

$$\sigma_s = 21.6\mu V \tag{8}$$

Result given in (8) is upper bound limit for absolute measurement error for one quadratic component (sine or cosine) of one single harmonic in spectra. So overall bound limit for absolute measurement error for one single harmonic is:

$$\sigma_{sH} = \sqrt{2}\cdot 21.6\mu V \approx 30.5\mu V \tag{9}$$

III. THEORY OF OPERATION

Starting from THD factor definition (1), THD_F can be measured with stochastic processor of orthogonal transformations. Since stochastic processor of orthogonal transformations obtains all of the harmonics of the input signal up to maximal order harmonic, THD_F can be measured directly. THD_F is fraction, so, overall measurement error can be calculated as the sum of dividend and divisor error. If we focus on THD_F measurement on signals in distributive power network, end in accordance with norm [1], it is enough to measure first 50 harmonics. Having (9), and taking in account all 50 harmonics, upper bound limit for absolute measurement error is

$$\sigma_{sH} = \sqrt{2\cdot 50}\cdot 21.6\mu V \approx 305\mu V \tag{10}$$

For fundamental frequency, upper bound limit for absolute measurement error is given with (9). Theoretically, maximal overall upper bound limit for absolute measurement error for THD_F factor is sum of (9) and (10) i.e. 335μV. Such large theoretical upper bound limit of absolute measurement error is far too pessimistic and not real. More close estimation of upper bound limit of absolute measurement error for THD_F factor is actually given with (10).

In order to obtain final measurement result, values from singular numerical accumulators must be squared and summed first. Then square root from overall sum should be calculated. Final step is to divide this number with fundamental harmonic amplitude. Block scheme of complete instrument is given in fig 3.

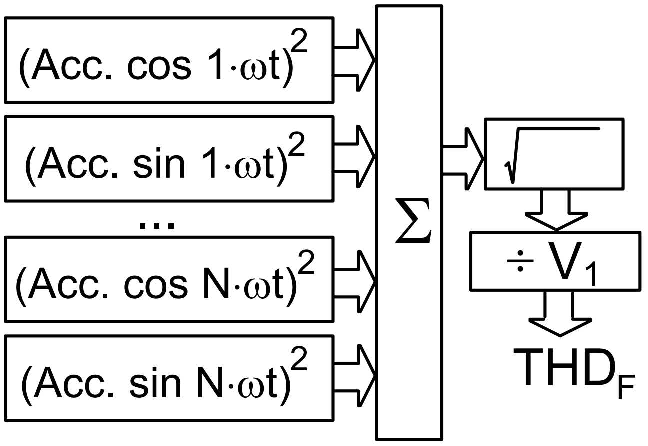

Fig. 3. Block scheme of the final instrument.

If it is acceptable to have smaller accuracy in THD factor measurement, it is possible to significantly reduce hardware complexity. Block diagram of simplified in hardware complexity, and a bit less accurate measurement device that utilizes stochastic approach is given in fig 4.

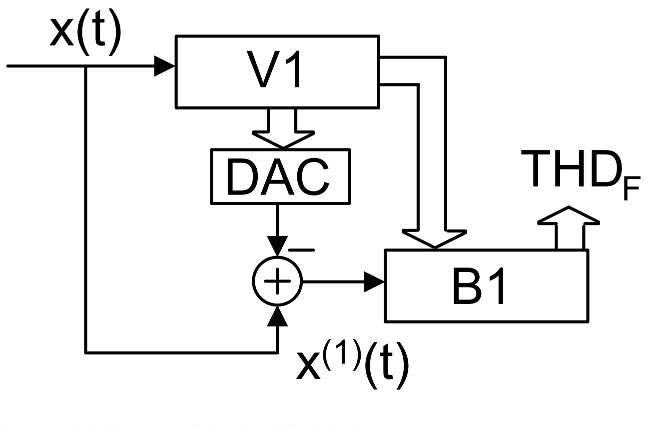

Fig. 4. Blok diagram of the simplified device

So, instead of measuring 50 harmonics with two components (sine and cosine) each, in fig 4 it is proposed solution where only fundamental harmonic is measured, but it is measured very accurately.

Then, during measurement process, fundamental harmonic is generated on D/A converter so, it appears back in measurement process as an analog signal. Then this signal is subtracted from original signal, so the analog difference contains all of the harmonics from original signal, apart from fundamental one. It means that all of the harmonics with order higher then one are together in this analog signal. In fig. 4, this signal is marked as $x^{(1)}(t)$. Further steps in THD measurement with the simplified instrument are:

– Measure true RMS of $x^{(1)}(t)$
– Measure true RMS of fundamental harmonic
– Calculate square root from $x^{(1)}(t)$ RMS
– Calculate quotient of square root from $x^{(1)}(t)$ RMS and fundamental harmonic value

With described procedure measurement is safe of spectral leakages.

IV. Simulation Results

Simulation verification is done on five different waveforms. Let's denote measurement signals with s_1, s_2, s_3, s_4 and s_5. Waveforms are given in fig. 5 to 9 respectively. On the left hand side of the each figure is shown waveform of given signal, while on the right hand side it is shown waveform of given signal without first harmonic.

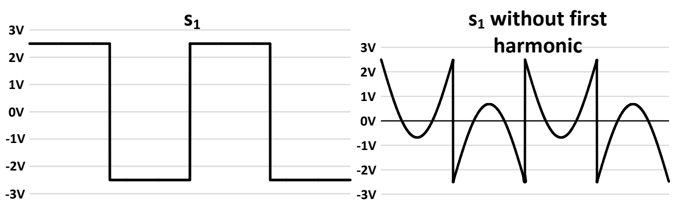

Fig. 5. Signal s_1, left – waveform, right – without fundamental harmonic

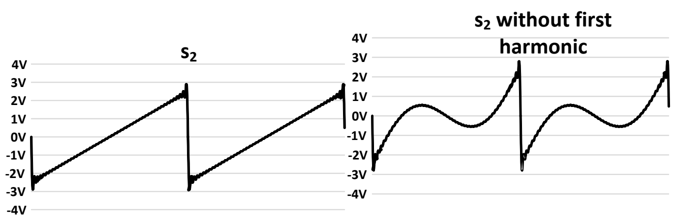

Fig. 6. Signal s_2, left – waveform, right – without fundamental harmonic

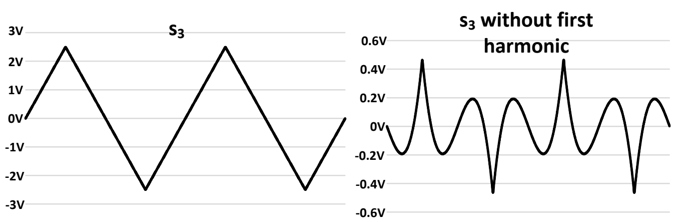

Fig. 7. Signal s_3, left – waveform, right – without fundamental harmonic

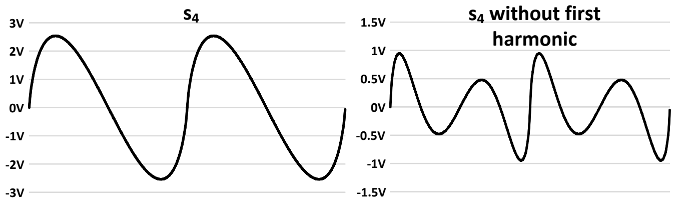

Fig. 8. Signal s_4, left – waveform, right – without fundamental harmonic

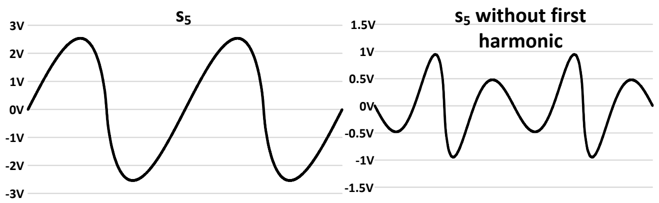

Fig. 9. Signal s_5, left – waveform, right – without fundamental harmonic

Now problem of THD factor measurement is reduced to measurement of true RMS of the signal without fundamental harmonic. Results of THD factor measurement as well as upper bound limit for absolute measurement are given in table 1.

signal	THD_F	Γ
s_1	47.82%	240µV
s_2	79.06%	221µV
s_3	2.42%	197µV
s_4	28.69%	238µV
s_5	28.69%	229 µV

Table 1. Simulation results

V. DISCUSSION

Precision and efficiency of proposed method depends mostly of precision and accuracy in measurement of fundamental harmonic in the signal. If fundamental harmonic is not measured with sufficient accuracy, new signal that is obtained as analog difference between input signal and its first harmonic will bring high measurement error in the dividend of THD result.

Stochastic method allows better accuracy and resolution then D/A converter for analog synthesis of the first harmonic. In that sense, it is desirable to optimize resolution for first harmonic measurement. If we take 14 bit resolution of D/A converter for analog synthesis of the first harmonic, and with 10 bit A/D converter in the input to measurement instrument, enough duration of the measurement is just one period of mains, i.e. 20ms for 50Hz mains. By subtracting in analog way first harmonic from the input signal, difference will not eliminate first harmonic entirely, but this difference error will be transferred in the measurement result. Proposed approach we can accept only if we accept lower accuracy in order to significantly decrease hardware complexity of proposed instrument.

VI. CONCLUSION

THD factor measurement is challenging metrological task. Since it is necessary to measure large number of harmonics, stochastic method is an optimal approach in THD factor measurement. In this paper, alternative approach is shown where first harmonic is subtracted from the input signal in analog circuit.

ACKNOWLEDGMENT

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REFERENCES

[1] IEEE Std 519-1992 IEEE Recommended Practices and Requirements for Harmonic Control in Electrical Power Systems, New York, NY: IEEE

[2] David Hong, Sanzhong Bai and Srdjan M. Lukić, "Closed-Form Expressions for Minimizing Total Harmonic Distortion in Three-Phase Multilevel Converters" IEEE TRANSACTIONS ON POWER ELECTRONICS, Vol. 29, No. 10, October 2014

[3] Avanish Tripathi and G. Narayanan “High-Performance off-line Pulse Width Modulation Without Quarter Wave symmetry for Voltage-source Inverter”, Advances in Electronics, Computers and Communications (ICAECC), 2014 International Conference on, Bangalore, India, 10-11 Oct. 2014, pp 1-6.

[4] Hadi Vadizadeh, Naeem Farokhniah, Hamidreza Toodeji and Ayoub Kavousi, “Formulation of line-to-line Voltage Total Harmonic Distortion of two-level Inverter With Low Switching Frequency”, IET Power Electronics,

[5] Shafiuzzaman K Khadem, Malabika Basu, Ruth Kerrigan and Biswajit Basu, “Power Quality Analysis of Energy Efficient Harmonic Loads”, Volume:6 , Issue: 3, pp 561 – 571, March 2013

[6] Shafiuzzaman K Khadem, Malabika Basu, Ruth Kerrigan and Biswajit Basu, “Power Quality Analysis of Energy Efficient Harmonic Loads”, 2014 IEEE Fourth International Conference on, Berlin, Germany, Sept. 2014, pp. 470 – 471.

[7] Viorel Apetrei, Constantin Filote and Adrian Graur, “Harmonic Analysis Based on Discrete Wavelet Transform in Electric Power Systems”, Kanjirapally, India, June 2013, pp 1-5.

[8] V. Pjevalica, V. Vujičić, "Further Generalization of the Low-Frequency True-RMS Instrument", IEEE Transactions on Instrumentation and Measurement, vol. 59, No. 3, March 2010, pp 736-744