Elastic F type Cable - Suspended Parallel Robot with One Mode

Mirjana Filipovic, *Member, IEEE*, Ljubinko Kevac, Ana Djuric

Abstract— **This paper is presents modeling approach of elastic F type Cable - Suspended Parallel Robot, eFCPR system with one mode. This topic requires studious approach and detection of specific phenomena, which needs to be defined through the process of mathematics modeling. The characteristic eFCPR structure is characterized by: shape of the work space, number of motors, number of hanging points, total number of ropes for the implementation of the CPR system, number of ropes from the camera carrier to the hanging points, type of motor, and type of winch. Each of these characteristics differently affects the response of the CPR system. The relations between the motor and the camera carrier motion are highly important for the kinematic and dynamic modeling of the eFCPR system. Two scenarios have been used for the eFCPR system analysis with one mode by OGIFLEX program package.**

Index Terms— **cable-suspended parallel robot, elastic ropes with one mode, kinematics, dynamics.**

I. INTRODUCTION

ENGINEERS and researchers started working on Cable-suspended Parallel Robot (CPR system) at the late 1980s and early 1990s. The first concepts of these systems have appeared in the United States and in Japan (cranes, parallel manipulators, cable-driven robotic systems). These systems became popular over time and they have become standard parts of today's robotics.

The presence of the elasticity in the cables of the CPR systems is presented for the first time in paper [1].

Our paper presents a more efficient model of the elastic CPR system in comparison with the previous research. The elastic characteristics of the CPR system have been introduced using the previous research [2-8], where the nonlinear dynamic elasticity has been defined. This experience has been used for the CPR dynamic modeling in order to define the camera position which is controlled with elastic ropes. These elastic ropes are modeled with the elastic characteristic in the axial direction. The elastic characteristic of the ropes cause the motion of the motors angular positions and motion of the camera position. The effect of the ropes axial elastic deformations are their transverse elastic deformations. By including the elastic characteristics of the ropes, the modeling of the CPR system became highly complex. This newly modeled system is named eFCPR system with one mode, elastic ropes F-type Cable-suspended Parallel Robot with one mode.

There is a detailed description of the eFCPR with one mode and its mathematical model in Section 2. The examples of the system responses for different conditions are analyzed in Section 3. The concluding remarks are presented in Section 4.

II. MATHEMATICAL FORMULATION OF EFCPR SYSTEM IN PRESENCE OF ONE MODE

The RFCPR system is modeled and analyzed by the ORVER software package. The system presented in paper [9] is used as a reference system for the development of an improved version of the eFCPR system presented in Figs. 1 and 2. The nature of this system requires development of a new methodology for calculation of its kinematics and dynamic models.

The mechanism was designed with three ropes. Each rope is connected to the related motor (blue rope is connected to the first motor, violet to the second motor, and red to the third motor). See Fig. 2. The synchronized motion of the ropes is done through the integrated (winches and motors) system. The ropes are elastic. Each of them has a one mode for all four directions of the hanging camera carrier. See Figs. 1 and 2. In each of these directions n_f, k_f, h_f and m_f there are one elastic element, i.e. one mode. Each elastic element is characterized by stiffness and damping. The connection between elastic deformations l_n, l_{k_1}, l_{k_2}, l_h, l_m of the directions n_f, k_f, h_f and m_f, and the angular positions of each motor θ_1, θ_2, θ_3 is defined by the new fictitious coordinates θ_{f_1}, θ_{f_2}, and θ_{f_3}.

$$\theta_{f1} = \theta_1 + \frac{l_n}{R_1} + \frac{l_{k1}}{R_1}. \tag{1}$$

$$\theta_{f2} = \theta_2 + \frac{l_h}{R_2} + \frac{l_{k2}}{R_2}. \tag{2}$$

$$\theta_{f3} = \theta_3 + \frac{l_m}{R_3}. \tag{3}$$

The complexity of the CPR system has been significantly increased by involving the elastic characteristics of the ropes. The number of DOF is increased too. The eFCPR system has 8 DOF. Any choice of generalized coordinates lids to accurate mathematical model of the selected system.

The relationship which connects velocities of the external coordinates $\dot{p} = [\dot{x} \quad \dot{y} \quad \dot{z}]^T$ with velocities of the fictitious

Mirjana Filipovic, Mihajlo Pupin Institute, University of Belgrade, Volgina 15, 11060 Belgrade, Serbia, e-mail: mirjana.filipovic@pupin.rs

Ljubinko Kevac, School of Electrical Engineering, The University of Belgrade, Bulevar Kralja Aleksandra 73, 11000 Belgrade, Serbia; Innovation center, School of Electrical Engineering, The University of Belgrade, Bulevar Kralja Aleksandra 73, 11000 Belgrade, Serbia, e-mail: ljubinko.kevac@gmail.com

Ana Djuric, Wayne State University , 4855 Fourth St. Detroit, MI 48202, U.S.A., e-mail: ana.djuric2@wayne.edu

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coordinates $\dot{\phi}_f = [\dot{\theta}_{f_1} \quad \dot{\theta}_{f_2} \quad \dot{\theta}_{f_3}]^T$ is defined in equation (4).

$$\dot{\phi}_f = J_{E_F} \cdot \dot{p}, J_{E_F} = \begin{bmatrix} J_{E11} & J_{E12} & J_{E13} \\ J_{E21} & J_{E22} & J_{E23} \\ J_{E31} & J_{E32} & J_{E33} \end{bmatrix}. \quad (4)$$

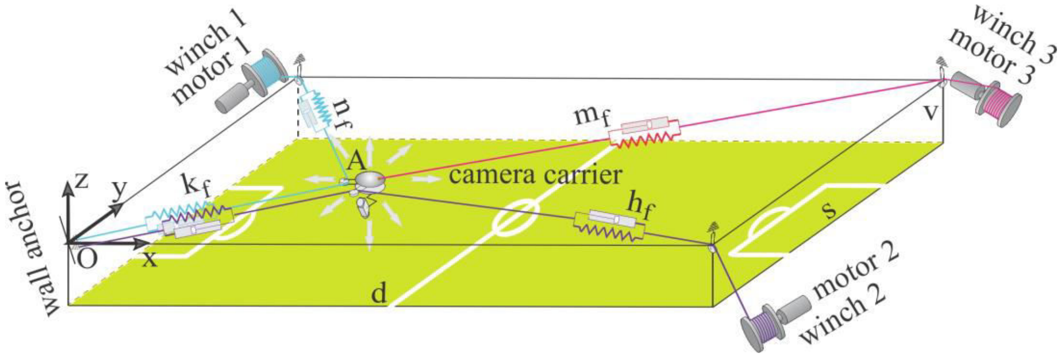

Fig. 1. eFCPR system, in 3D space.

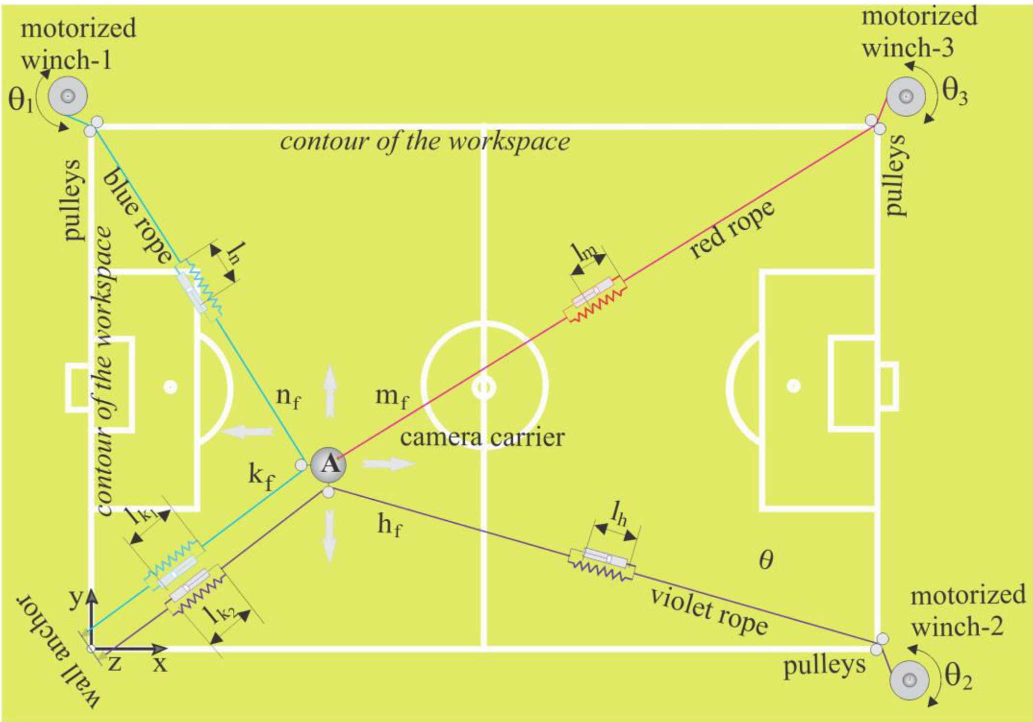

Fig. 2. eFCPR system, in 3D space.

The kinematics model has been developed using a Jacobian matrix J_{E_F}. This novel procedure is named the Kin-eFCPR-Solver which means Kinematics elastic F-type Cable Parallel Robot Solver. The eFCPR system has three motors, and its mathematical model is expressed with vector equation (5):

$$u = G_v \cdot \ddot{\phi} + L_v \cdot \dot{\phi} + S_v \cdot M_R. \quad (5)$$

The relation between the resultant load torque $M_R = [M_1 \quad M_2 \quad M_3]^T$ and the elastic load torque $M_E = [F_{En} \cdot R_1 \quad F_{Eh} \cdot R_2 \quad F_{Em} \cdot R_3]^T$ is defined with the vector equation (6).

$$M_R = -M_E. \quad (6)$$

By substituting vector equation (6) into vector equation (5) the vector equation is generated:

$$u = G_v \cdot \ddot{\phi} + L_v \cdot \dot{\phi} - S_v \cdot M_E \quad (7)$$

The Lagrange principle of virtual work has been used to find the relation between the elastic load torque M_E and the camera driving force $F_p = [F_x \quad F_y \quad F_z]^T$.

$$(M_E)^T \cdot \dot{\phi}_f = F_p^{\ T} \cdot \dot{p}. \quad (8)$$

By substituting vector equation (4) into vector equation (8), the resulting vector equation is independent of $\dot{p}$. After it transposes, the final result is shown with vector equation (9):

$$M_E = O_E \cdot F_p. \quad (9)$$

The torque mapping matrix $O_E = \left(\left(J_{E_F}\right)^T\right)^{-1}$ represents a strong coupling between the presented motors. The eFCPR system modeling includes three motors which dynamics is described with three equations by substituting vector equations (9) into vector equation (7). The resulting equation (10) is in the vector form too.

$$u = G_v \cdot \ddot{\phi} + L_v \cdot \dot{\phi} - S_v \cdot O_E \cdot F_p . \qquad (10)$$

To define the complete dynamic model, more two equations need to be added. Those two equations describe the equilibrium k_f direction, in both ropes.

$$F_{Ek_1} = F_{Ek_2} = \Diamond \cdot [\frac{x}{k_f} \quad \frac{y}{k_f} \quad \frac{z}{k_f}] \cdot F_p . \qquad (11)$$

The equations (11) represents the relation between the elastic force F_{Ek_1} (F_{Ek_2}) and the camera driving force F_p using factor $\Diamond = 0.5$ analogously to [10]. This procedure for the RFCPR system is explained in details in paper [9]. From previous calculations it is clear that the eFCPR mathematical model has been described with eight equations, which implies that the eFCPR system has eight DOF. These equations are vector equations (9) and (10), and two scalar equations (11). These equations represent the dynamic model of the eFCPR system. The selected control law is a classical PD controller and it is described in the following vector equation:

$$u_i = K_{lpi} \cdot (\theta_i^o - \theta_i) + K_{lvi} \cdot (\dot{\theta}_i^o - \dot{\theta}_i) . \qquad (12)$$

III. Simulation Experiments - Program Package OGIFLEX

In this paper, the mathematical model of eFCPR system with one mode has been analyzed and synthesized using program package OGIFLEX, see [11]. The software package OGIFLEX is used for validation of the applied theoretical contributions. The software package OGIFLEX includes three essential modules, which are the kinematics, dynamic and motion control law solvers for the eFCPR system.

Two case studies were analyzed and presented. The first Example is implemented with winches' radius: $R_1 = R_2 = R_3 = 0.15(m)$ and second Example with radius: $R_1 = R_2 = R_3 = 0.09(m)$.

All other parameters are the same. The motors are selected by Heinzman SL100F and the gear boxes are HFUC14-50-2A-GR+belt.

The camera carrier moves in the 3D space and its velocity has a trapezoidal shape $v_{\max}^o = 0.4167(m/s)$.

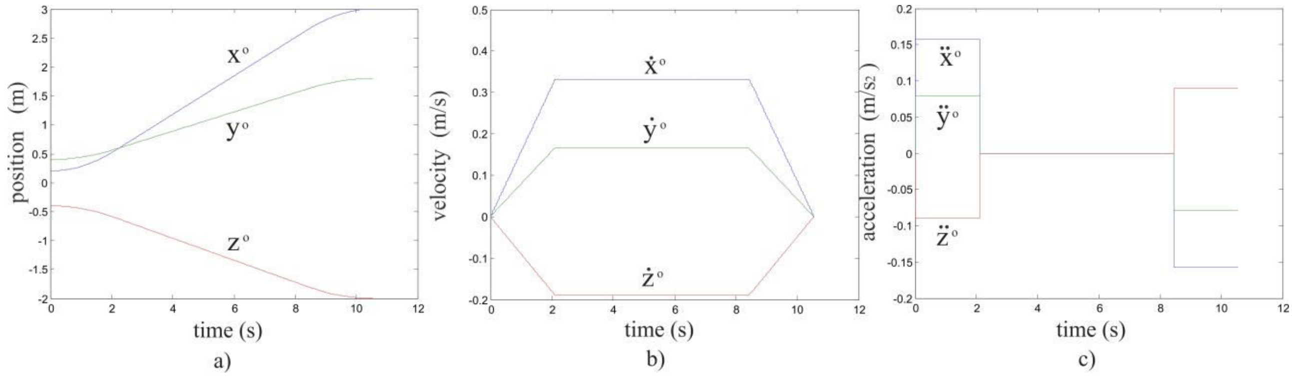

Fig. 3. The reference trajectory of the camera carrier a) position, b) velocity, c) acceleration.

The starting point is $p_{start}^o = [0.2 \quad 0.4 \quad -0.4](m)$, and the end point $p_{end}^o = [3.0 \quad 1.8 \quad -2.0](m)$. See Fig. 3.

There are in total five different elastic elements: l_n , l_{k_1} , l_{k_2} , l_h , l_m in n_f , k_f , h_f and m_f directions. Each of the five elements naturally has the stiffness characteristic $C = 7.0 + 004(N/m)$ and damping characteristic $B = 7.0 + 006(N/(m/s))$.

To avoid confusion in choosing the stiffness and damping characteristics, we have introduced a little simplification in this paper: It is assumed that the ropes rigidity and damping elastic characteristics are the same for all elastic ropes.

Figures 4-5 represent the results for selected case studies (Examples 1-2).

Each Example has nine graphs related to:

a) the camera carrier position at the reference and the real frames,

b) the motor shaft position at the reference and the real frames,

c) the forces in the ropes at the reference and the real frames,

d) the deviation between the real and the reference trajectory of the camera carrier,

e) the deviation between the real and the reference trajectory of the motor shaft positions,

f) the reference and the real control signals,

g) the elastic deformations l_n , l_h , l_m in n_f , h_f , m_f directions, respectively at the real frame and $l_n^o = l_h^o = l_m^o = 0$ at the reference frame,

h) the elastic deformations l_{k_1} , l_{k_2} ($l_{k_1} = l_{k_2}$) in k_f direction, and $l_{k_1}^o = l_{k_2}^o = 0$ at the reference frame,

i) the elastic forces F_{En} , F_{Ek_1} , F_{Ek_2} ($F_{Ek_1} = F_{Ek_2}$), F_{Eh} , F_{Em} at the real frames only.

Example 1: The first Example is performs with winches' radius: $R_1 = R_2 = R_3 = 0.15(m)$ and shows a good response. The tracking error of the motor's angular positions and the camera carrier's motion with respect to the desired values are in the acceptable range and the eFCPR system does not have an oscillatory response. See Fig. 4.

Example 2: Decreasing the winches' radius to value of: $R_1 = R_2 = R_3 = 0.09(m)$ the system responses have changed. In that case the tracking error of the desired trajectory is much worse. In this case the system response is worse, compare with Example 1 from Fig. 4.

First motor control signal enters the saturation at the $4.7(s)$ and stay in saturation until $10.2(s)$. It is clear that in this time period the tracking of the motor angular positions is unsatisfactory.

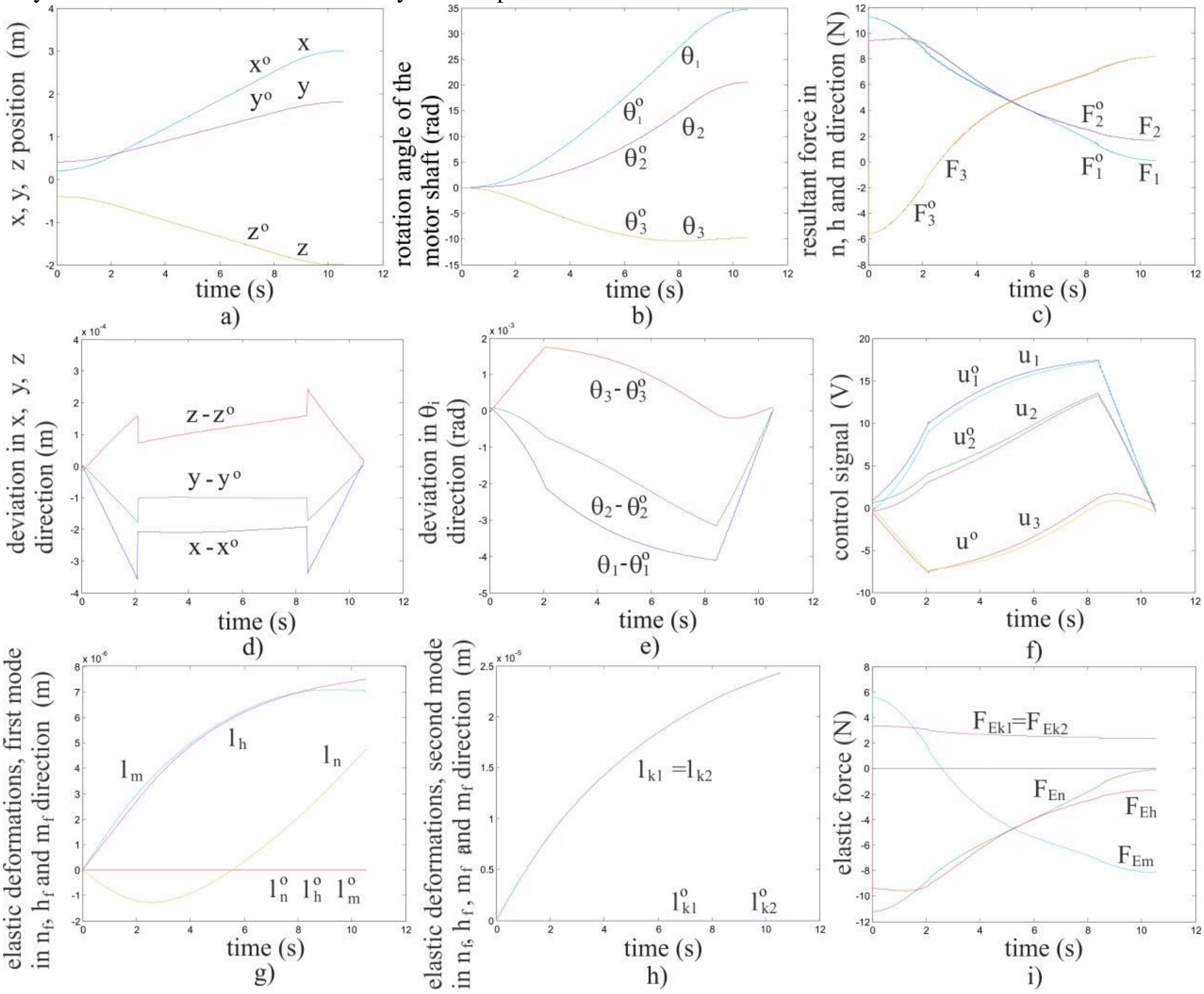

Fig. 4. Example 1.

This indirectly causes poor tracking of the camera carrier's desired trajectory in x, y and z directions of the Cartesian space. See Fig. 5.

This analysis shows that the motion dynamics of each motor depends significantly on the eFCPR parameters selection.

These examples prove the response system which depends on the winches' radius; rope's stiffness and damping characteristics; trajectory selection and other system characteristics.

The eFCPR system is modeled and analyzed by using the program package OGIFLEX.

IV. Conclusion

This paper presents a novel methodology for the eFCPR system kinematics and dynamic modeling. The camera carrier is supported by ropes whose elasticity is characterized with one mode. The key element in the eFCPR system modeling is the relation between the elastic deformations of the ropes and the motors' angular positions. This novel procedure uses the fictitious coordinates and is named ED+M method, which means Elastic Deformations plus Motor motion. The kinematics model is defined via the Jacobian matrix. This methodology is named Kin-eFCPR-solver and it represents a guideline for solving kinematics structure of the eFCPR system types. The relation between the elastic load torque and the forces at the camera carrier is described by the Lagrange principle of virtual work. The dynamic characteristics of the motors are an integral part of the eFCPR system's mathematical model.

The highly authentic solution of the eFCPR system model has been defined.

The desired trajectory of the rigid system RFCPR has been used as the reference trajectory of the elastic system eFCPR. The RFCPR system is modeled and analyzed by the ORVER software package. In the control design of the eFCPR trajectory tracking the only disturbance is the elastic characteristic of the ropes, which are unknown at the reference trajectory.

Two scenarios have been used for the eFCPR system analysis. The elastic deformations of the ropes cause nature behavior of the camera carrier's motion in 3D space. The software package OGIFLEX has been developed and used for the individual and comparative analysis of the eFCPR system with one mode.

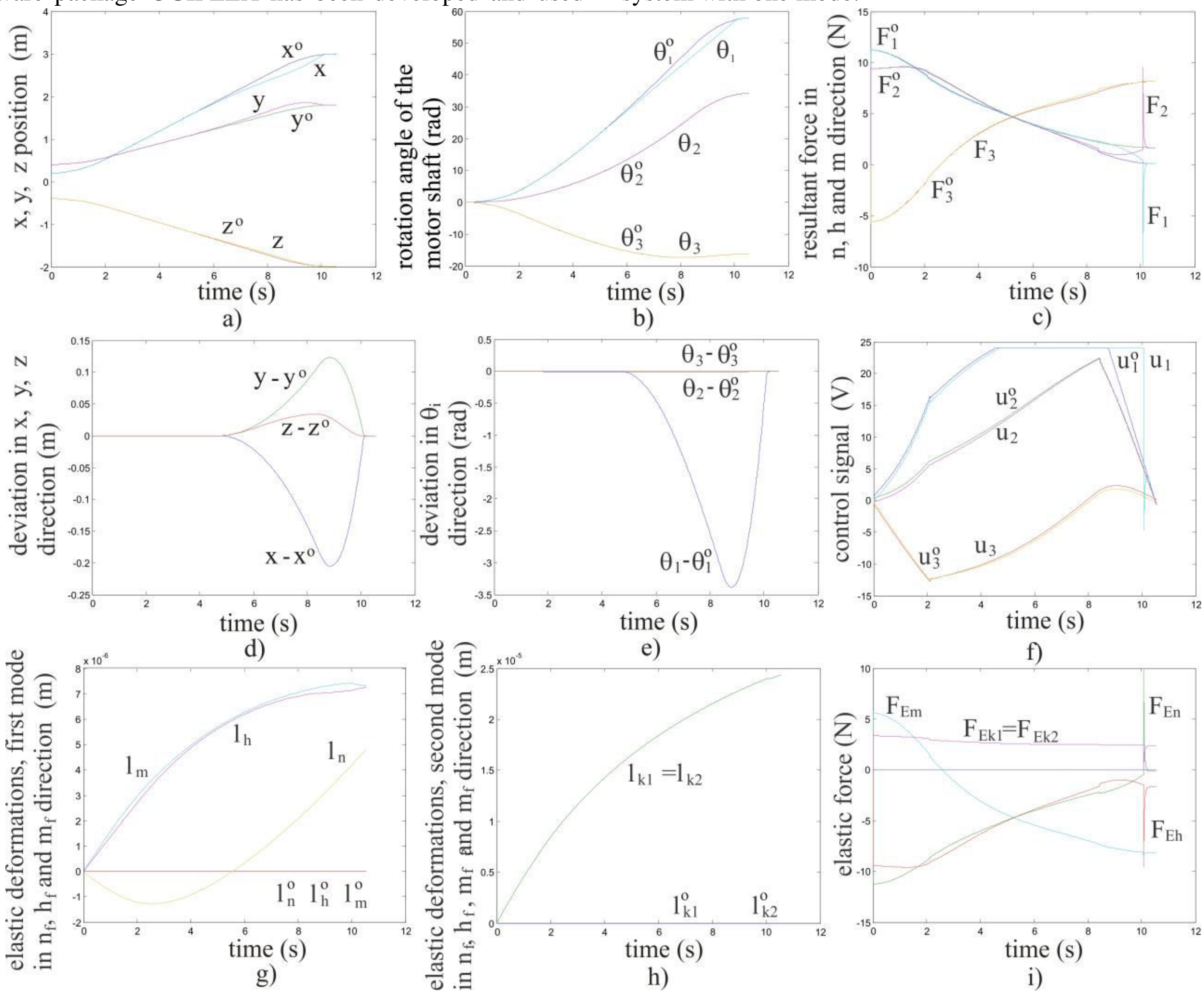

Fig. 5. Example 2.

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