

# On the transmission of the halftoned image over Double-Weibull channel

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**Abstract**—In this paper, FSO (Free-Space Optical) transmission of the halftoned image over the Double-Weibull turbulence channel is analyzed. First, halftoning method, and algorithm for FSO transmission simulation are explained. Then, transmission of halftoned image is described. At the reception side, we have used an average bit error rate (ABER) for reconstructed image performance measure, as a function of FSO Rytov variance. Obtained simulation results are shown in tables and graphically presented. Finally, performance analysis has been carried out, through comparing results as a function of Rytov variance (in strong and moderate fluctuation regime).

**Index Terms**— Halftone image; FSO transmission; Double - Weibull.

## I. INTRODUCTION

Multimedia content is very present in everyday communication, since distribution of multimedia content such as newspapers and books and is very present in all forms of digital communication. Some of produced images are produced over the halftoning and they contain only two colors, black and white. When this halftone image is viewed from a certain distance, it appears as the original image [1]. Halftoning methods developed in past years can be categorized into ordered dithering [2], error diffusion [3], [4], dot diffusion [5], [6] and direct binary search [7], [8].

Usage of high-data-rate free-space optical (FSO) transmission systems has grown recently, since FSO transmission obtains many advantages over radio frequency (RF) transmission, including: power concentration in narrow beam, immunity to electro-magnetic interference, no need for Fresnel zone obtainment, full duplex transmission possibility, small practical realization complexity, license-free transmission. Main advantages of FSO transmission are caused by short dimension of wavelengths used for optical transmission. However, there are some disadvantages that occur during transmission, such are: acquisition and pointing are more difficult, various drawbacks arise due to influence of atmospheric factors (i.e. haze, fog, rain, Sun, turbulence). Major performance impairment in FSO links, which degrades

the link performance is atmospheric turbulence-induced fading [9]. Temporal and spatial fluctuations of the laser beam, that occur as a result of variations in the refractive index, which are caused by atmospheric turbulence, manifest as irradiance fluctuations in the received signals. These phenomena is known as FSO fading or scintillation. In FSO communication, broadcast of digital image is very intense. Since typical FSO fade could last few ms, for a FSO link operating at the speed of several Gb/s, large number of consecutive bits could be inaccurately transmitted or even lost, so receiver signal would be attenuated, which would result in higher bit error ratio (BER).

In order to obtain accurate modeling of FSO propagation, various mathematical and numerical models, have been provided in the literature. Weibull distribution model is mostly used for modeling of scintillation of signal with different intensities of turbulences and is often applied in systems with large aperture on the receiving side [10]. Log - Normal distribution model [9] is used for modeling scintillation related to the regimes of weak atmospheric turbulence [11]. Rician distribution model is used for description of scintillation that occurs in terrestrial communication channels in sparsely populated areas and suburbs [12]. Gamma-Gamma distribution is a very simplified model of scintillation that can be applied for other regimes of turbulences [13]. However, recently Double Generalized Gamma (DGG) turbulence model [14], [15] has been considered since it generalizes many existing turbulence channel model and provides an excellent fit to the plane and spherical waves simulation data. This model can be reduced to other previously mentioned models (except for the Rician turbulence model) of scintillation by setting the corresponding values to the DGG model parameters. With setting  $\gamma_{i,n} = 1$  and  $\Omega_{i,n} = 1$ , DGG can be presented as Gamma distribution model, with setting  $m_{i,n} = 1$ , can be presented as Double-Weibull model [16], and when is set  $\gamma_{i,n} = 1$ ,  $\Omega_{i,n} = 1$  and  $m_{2,n} = 1$  it can be recognized as  $K$  distribution model. One more generic option is when set  $\gamma_{i,n} \rightarrow 0$  and  $m_{i,n} \rightarrow \infty$  this model can be recognized as Log-Normal distribution model. Instead of the randomly generated bits, an halftoned image was used as a source of information (halftoned black-and-white image) and the performance of the FSO system is analyzed.

In this paper performance analysis of halftoned image FSO transmission through Double-Weibull channel was carried out. The paper is composed in the following way: section II describes halftoning method and Double-Weibull FSO channel model. The obtained simulation results and obtained

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performances analysis are presented in section III. Concluding remarks are given in section IV.

## II. SYSTEM MODEL

Received FSO signal field at the aperture plane of the receiver can be modeled as:

$$E_r(t, r) = u_s(t) \exp(j2\pi f_c t + \theta(t)) \exp[\chi(r) + j\phi(r)] \quad (1)$$

where  $r$  denotes the position vector on the receive aperture plane,  $f_c$  is the optical carrier frequency, and  $u_s(t) \exp(j\theta(t))$  is the complex envelope of the modulation signal. Here,  $\chi(r)$  and  $\theta(r)$  represent the turbulence-induced amplitude fluctuations and phase variations of the channel.

Further, FSO signal at the output of photodetector receiver can be modeled as:

$$y_T(t) = x_T(t) + n_T(t) \quad (2)$$

where  $n_T(t)$  denotes total noise obtained as a sum of thermal and shot noise variances expressed as:

$$\sigma_n^2 = \sigma_{th}^2 + \sigma_{sh}^2 = 4k_B \frac{T}{R_L} F_n \Delta f + 2qg^2 F_A R P_t m I \Delta f \quad (3)$$

where  $\sigma_{th}^2$  - denotes thermal noise and  $\sigma_{sh}^2$  - denotes the shot noise and where  $k_B$  denotes Boltzmann constant and  $F_n$  denotes amplifier noise figure,  $T$  and  $R_L$  are temperature and load resistance respectively,  $\Delta f$  is the effective noise bandwidth. The effective bandwidth is dependent on the bit rate,  $R_b$ , as  $\Delta f = R_b/2$ ,  $q$  represents an electron charge,  $g$  and  $R$  represent gain and responsivity, respectively,  $P_t$  denotes transmitted optical power,  $m$  denotes modulation index,  $I$  represent accounted normalized irradiance, and  $F_A$  denotes the excess noise factor presented in [17].

Information carrying part of signal can be modeled as:

$$x_T(t) = \frac{e\eta\pi}{2hf_c} AD^2 u_s(t) \text{Re}\{\alpha \exp(j2\pi f_{IF} t + \theta_{IF}(t))\} \quad (4)$$

where  $f_{IF} = f_c - f_{LO}$  is the equivalent signal frequency and  $\alpha$  represents the effective FSO fading fluctuation modeling the channel.

The PDF of the FSO fading amplitude modeled with Double Generalized Gamma distribution is modeled as:

$$f_i(I) = \frac{\gamma_2 p p^{m_2-1/2} q^{m_1-1/2}}{\Gamma(m_1) \Gamma(m_2) (2\pi)^{\frac{p+q-1}{2}}} I^{-1} \times G_{p+q,0}^{0,p+q} \left[ \left( \frac{\Omega_2}{I^{\gamma_2}} \right)^p \frac{p^p q^q \Omega_1^q}{m_1^q m_2^p} \middle| \Delta(q:1-m_1), \Delta(p:1-m_2) \right], \quad (5)$$

where  $G_{p,q}^{m,n}(x)$  is the Meijer G-function defined in [18],  $\Gamma(x)$  denotes special Gamma function [18],  $p$  and  $q$  are

positive integer numbers that satisfy  $p/q = \gamma_1/\gamma_2$ , and  $\Delta(j;x) = x/j, \dots, (x+j-1)/j$ . Parameters  $\gamma_1$ ,  $\gamma_2$ ,  $\Omega_1$ ,  $\Omega_2$ ,  $m_1$ ,  $m_2$ , are parameters of Generalized Gamma distributions, which model statistically independent random processes arising respectively from largescale and small scale turbulent eddies. These parameters can be identified using the moments of small and large scale irradiance fluctuations and are directly tied to the atmospheric parameters as shown in [14]. Parameters  $\Omega_1$  and  $\Omega_2$  thus can be expressed through the parameters  $\sigma_x$  and  $\sigma_y$ , the variances of small-scale and large-scale fluctuations, directly tied to the atmospheric conditions.

Setting  $m_1 = 1$  and  $m_2 = 1$  this DGG model will be presented as Double-Weibull model and PDF will be modeled as:

$$f_i(I) = \frac{\gamma_2 p (pq)^{1/2}}{(2\pi)^{\frac{p+q-1}{2}}} I^{-1} G_{p+q,0}^{0,p+q} \left[ \left( \frac{\Omega_2}{I^{\gamma_2}} \right)^p p^p q^q \Omega_1^q \middle| \Delta(q:0), \Delta(p:0) \right] \quad (6)$$

Assuming a plane wave when inner scale effects are considered, the variances for the large-scale and the small-scale scintillations are given by [14] in the forms of the ratio of Fresnel zone to finite inner scale and Rytov variance ( $\sigma_{Rytov}^2$ ):

$$\sigma_{Rytov}^2 = 1.23 C_n^2 k^{7/6} L^{11/6} \quad (7)$$

where  $k = 2\pi/\lambda$  is the wave-number,  $\lambda$  is the wavelength,  $L$  - propagation distance and  $C_n^2$  refraction index.

Similarly, assuming a spherical wave in the absence of inner scale, the variances for the large-scale and the small-scale scintillations are given by [19].

Received instantaneous Signal-to-Noise Ratio (SNR) of the system after demodulation is given as:

$$\gamma = \frac{P_x}{\sigma_n^2} = \frac{P_x}{2(\sigma_{th}^2 + \sigma_{sh}^2)} \quad (8)$$

where  $P_x$  is the output signal power.

Algorithm for simulation of FSO halftoned image transmission is accomplished in the following steps:

*Step 1:* From halftoned digital image  $A^*$  (8-bit's) is created vector. *Step 2:* Obtained vector, with decimal elements, is translated to vector with binary elements  $D$  dimension  $24 \times M \times N$ . *Step 3:* On vector  $D$  is applied Binary Phase Shift Keying (BPSK) modulation  $X = 2 \cdot D - 1$ . BPSK is modulation scheme that conveys data by changing the phase of a carrier wave by using two phases which are separated by  $180^\circ$ . This modulation is the most robust of all the PSKs since highest level of noise or distortion is necessary to make the demodulator reach an incorrect decision. *Step 4:* The BPSK modulated signal is transmitted through the Ricean turbulence channel:  $Y = H * X + N$ , with  $H$  being vector that encompasses the influence of Ricean fading, whose samples are generated according to Eq. (5), with respect to the parameter  $K$ , the ratio of the strength of the coherent component, defined as in Eq.(6). Vector  $N$  encompasses the channel AWGN of defined

SNR level. *Step 5*: Grey decoding with hard decisioning has been provided on  $Y$ , and vector  $\overline{D}$  has been obtained. *Step 6*: Resulting halftoned digital image  $\overline{A^*}$  vector is obtained from vector  $\overline{D}$ .

### III. SIMULATION RESULTS AND PERFORMANCE ANALYSIS

In order to simulate FSO transmission of halftoned images over the Double-Weibull turbulence fading channel following experiment is conducted:

*Step 1*: Original image is halftoned with *Error Diffusion method*. An error diffusion algorithm [4] is defined by the following steps: 1) Adding to each pixel location, at the input picture,  $I(i)$ , a weighted average of the previous errors in some neighborhood in order to obtain the modified input  $M(i)$ . 2) Choosing output image pixel values  $O(i)$  as an element of output pixels vector  $V$  closest to an element of closest to  $M(i)$ . 3) Defining the error  $e(i)$  as  $M(i) - O(i)$ . 4) bounding of error at defined value. 5) For used 8bit image, threshold is defined at 127.5. *Step 2*: On halftoned image  $A^*$  is applied BPSK modulation. *Step 3*: BPSK modulated signal has been carried through Rician turbulence fading channel with AWGN. *Step 4*: Image has been reconstructed at the reception after performing Gray decoding with hard decisioning. *Step 5*: From decoded signal is reconstructed transmitted halftoned image.

As the quality measure of transmitted halftone image, the Mean Square Error (MSE), Peak Signal-To-Noise Ratio (PSNR), Normalized Correlation (NC) and Bit Error Rate (BER) are considered.

$$MSE = \frac{\sum_{ij} (x_{ij} - y_{ij})^2}{M \times N} \quad i = 1 \dots M, \quad j = 1 \dots N \quad (9)$$

$$PSNR = 10 \log_{10} \frac{2^n - 1}{MSE} \quad (10)$$

$$NC = \frac{\sum_{ij} (x_{ij} y_{ij})}{\sqrt{\sum_{ij} (x_{ij})^2} \sqrt{\sum_{ij} (y_{ij})^2}} \quad i = 1 \dots M, \quad j = 1 \dots N \quad (11)$$

$$BER = \frac{\sum_{ij} [(x_{ij})_l]_2 \oplus [(y_{ij})_l]_2}{M \times N} \quad i = 1 \dots M, \quad j = 1 \dots N, \quad l = 1 \dots n \quad (12)$$

where:  $x_{ij}$  is pixel of original image,  $y_{ij}$  is pixel of restored image,  $M \times N$ - the size of the image, and  $\oplus$  denotes EXOR operator over each of  $n$  pair of bits from  $x_{ij}$  and  $y_{ij}$ .

Original images (dimensions 512x512) presented on Fig. 1 presents the image's base for the experiment: a) Lena, b) Girl, c) Baboon, d) Peppers. Images presented on fig. 2 are halftoned images obtained by using error diffusion method [4]. The values of Rytov coefficient are varied in the range  $\sigma_{Rytov}^2 = 1.5 - 5.5$ .

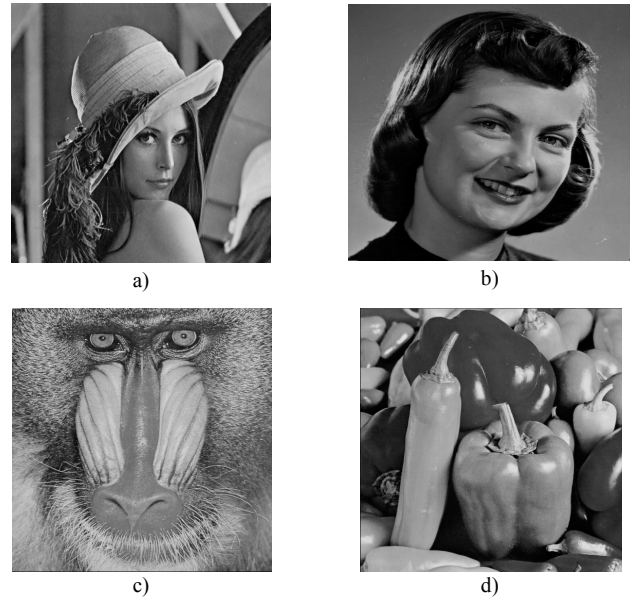


Fig 1. Images used in the paper: a) Lena, b) Girl, c) Baboon and d) Peppers

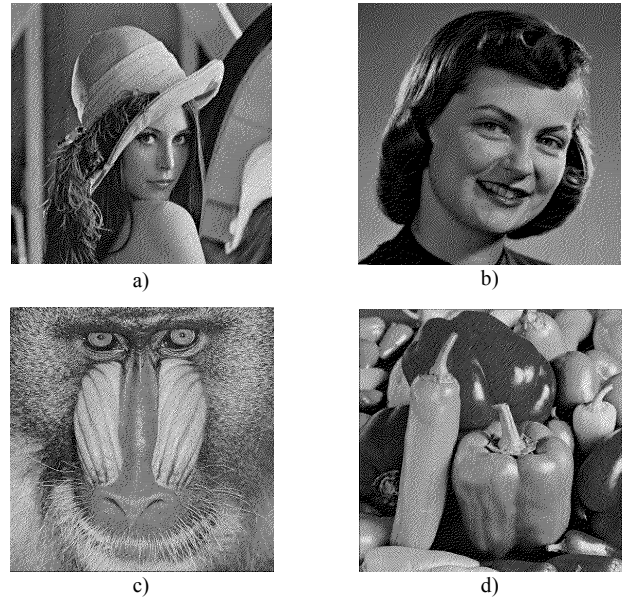


Fig 2. Halftoned images used in the paper: a) Lena, b) Girl, c) Baboon and d) Peppers

Reconstructed image (Lena) after transmission through Double-Weibull FSO channel are presented at Fig. 3. Quality measure values for observed images are given on Fig. 4 and in Table I.

TABLE I. QUALITY MEASUREMENTS  
MSE, PSNR AND NC FOR LENA

| $\sigma_{Rytov}^2 = 1.5 - 5.5$ | MSE    | PSNR  | NC    |
|--------------------------------|--------|-------|-------|
| 1.5                            | 0.09   | 58.43 | 0.998 |
| 2                              | 0.053  | 60.87 | 0.998 |
| 2.5                            | 0.2669 | 53.87 | 0.994 |
| 4.5                            | 0.122  | 57.25 | 0.997 |
| 5                              | 0.143  | 56.57 | 0.997 |
| 5.5                            | 0.329  | 52.95 | 0.993 |



a)



b)

Fig 3. Transmitted halftoned image Lena with parameter  $\sigma^2_{Rytov}$  : a)  $\sigma^2_{Rytov} = 2$ , and b)  $\sigma^2_{Rytov} = 5$ .

After analyzing MSE, PSNR and NC values obtained for the halftoned image, transmitted through Double-Weibull turbulence channel (presented in Table I), we can see that the transmitted image is with much better quality in moderate regime with  $\sigma^2_{Rytov} = 2$ , then in strong turbulence regime,  $\sigma^2_{Rytov} = 5$ . In this regime quality MSE is ten times higher (better) then in strong turbulence regime, also and PSNR and NC values are beter. Further performance improvement could be obtained by designing FSO link parameters (optical link distance, transmitted signal power, optical bandwidth, carrier

signal wavelength, aperture diameters at the transmitter / receiver).

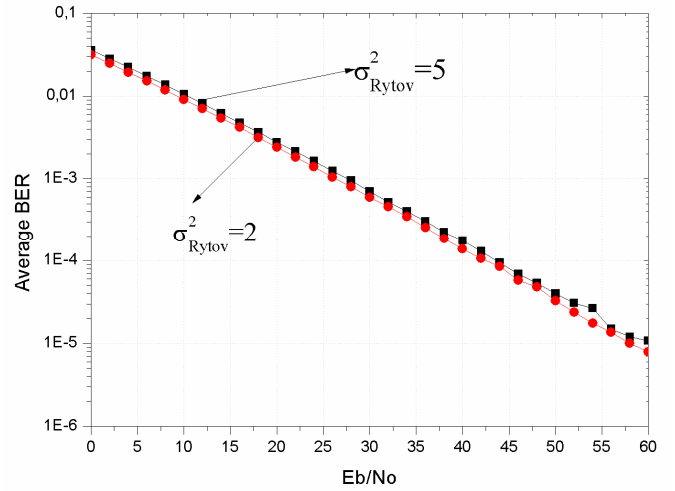


Fig 4. Quality measurement BER for different value of  $\sigma^2_{Rytov} = 2$  and 5.

#### IV. CONCLUSION

Performances of the FSO transmission of the halftoned image over the Double-Weibull turbulence channel has been considered through the prism of received PSNR, MSE and NC values. It has been shown that for FSO channel parameter value  $\sigma^2_{Rytov} = 2$  quality measures MSE and PSNR are ten time higher (better) then in analyzed strong turbulence regime, with  $\sigma^2_{Rytov} = 5$ . Capitalizing on proposed method FSO link parameters could be efficiently determined in the FSO link designing process, in order to reach demanded MSE and PSNR quality of halftoned image at the reception.

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