Calibration of Local Oscillators Mismatch in a Multi-channel Receiving System

Miljko M. Erić, Nenad J. Vukmirović, and Miloš T. Janjić

Abstract—**In this paper we propose a procedure for calibration of local oscillator mismatch in a collocated multi-channel receiving system. It is based on a new algorithm for joint relative phase and frequency offset estimation successively applied to pairs of channels, and compensation of oscillator mismatch using these estimated parameters. It is assumed that the receiving channels are time synchronized. The theoretical Cramer-Rao Bound (CRB) for phase and frequency offset estimation is derived. Results of simulations are compared with the CRB. Performance of the proposed calibration procedure is also illustrated by experimental results using software defined radio platforms.**

***Index Terms*—phase and frequency offset estimation, oscillator mismatch, multi-channel receiver, beamforming, antenna arrays, spatio-temporal spectrum sensing, distributed MIMO**

I. Introduction

THE key challenge in implementing array processing algorithms in any field of applications such as in direction finding, beamforming, adaptive antenna arrays, bistatic radars, astronomy, etc, is related to the problem of matching of channels in a multichannel receiving system with collocated or distributed receivers. Regardless of the applied technology, calibration of channel mismatches is needed. There are many elements that contribute to mismatch between channels but in practice phase and frequency offset of local oscillators is the key problem to be solved in a calibration procedure. Recently, calibration of local oscillators mismatch has been the focus of researches in the fields of spatio-temporal spectrum sensing [1], distributed beamforming [2] and distributed MIMO systems [3]. A procedure for calibration of phase and frequency offsets in a collocated multichannel receiving system is proposed in this paper. Novelty in our proposed procedure is a new algorithm for joint phase and frequency offset estimation as well as derived a closed form analytic expression of the Cramer-Rao bound (CRB) for the estimation problem.

Miljko. M. Erić is with University of Belgrade, School of Electrical Engineering, Bulevar Kralja Aleksandra 73, 11020, Belgrade, Serbia (e-mail: miljko.eric@etf.rs).

Nenad J. Vukmirović is with University of Belgrade, School of Electrical Engineering, Bulevar Kralja Aleksandra 73, 11020, Belgrade, Serbia (e-mail: vn135023p@student.etf.rs).

Miloš. T. Janjić is with University of Belgrade, School of Electrical Engineering and Innovation Centre of School of Electrical Engineering, Bulevar Kralja Aleksandra 73, 11020, Belgrade, Serbia (e-mail: milos.janjic@ic.etf.rs).

II. Problem Formulation

Let us consider a multichannel receiving system with L receiving channels, Fig. 1. At every receive channel, input time multiplexing of a calibration signal and a data signal received from the antenna is performed according to Fig. 1. External frequency reference signal is applied to every receive channel by means of a splitter. The cables from the splitter to the receivers are calibrated. Internal local oscillators in the receive channels lock to this reference signals by using their PLLs. Unfortunately, relative frequency and phase offsets between signals at the output of PLLs are present. Since array processing applications require perfect frequency, as well as time synchronization of the receiving channels, relative phase and frequency offsets have to be estimated and compensated. Suppose that the receiving channels are perfectly time synchronized, so that the phase offset corresponds to the beginning of the observation interval, $T_C + T_A$, and the frequency offset is constant during the interval. For simplicity, we assume unit channel gains. If so, the phase and frequency offset in the time interval T_A can be compensated for by estimation of these offsets using the calibration signal received in the time interval T_C. In the receiving channels, the signal is IQ down converted to baseband and then time synchronous AD conversion of I and Q signals takes place, so that time samples of the signal complex envelope of each receive channel are available for further processing. Complex envelopes in the receiving channels contain relative frequency and phase offsets due to unadjusted local oscillators in the channels. Discrete time baseband mathematical model of the signal received by a multichannel receiving system in the time interval T_C can be formulated in matrix form as:

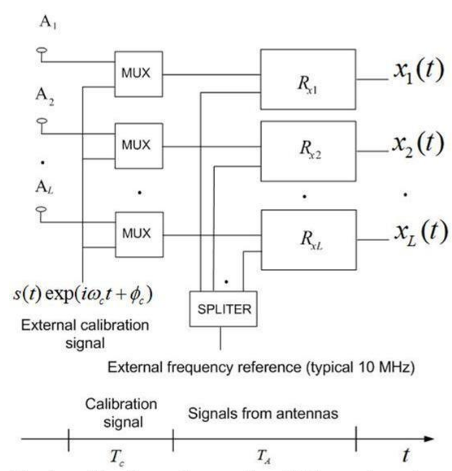

Fig.1. calibration scheme of multichannel receiver

$$\underbrace{\begin{bmatrix} x_1(n) \\ x_2(n) \\ \cdot \\ x_L(n) \end{bmatrix}}_{\mathbf{X}\in C^{Lx1}} = \begin{bmatrix} e^{j\varphi_1}e^{j\omega_1 n} \\ e^{j\varphi_2}e^{j\omega_2 n} \\ \cdot \\ e^{j\varphi_L}e^{j\omega_L n} \end{bmatrix} s(n) + \begin{bmatrix} \eta_1(n) \\ \eta_2(n) \\ \cdot \\ \eta_L(n) \end{bmatrix} = \underbrace{\begin{bmatrix} 1 \\ e^{j\Delta\varphi_2}e^{j\Delta\omega_2 n} \\ \cdot \\ e^{j\Delta\varphi_L}e^{j\Delta\omega_L n} \end{bmatrix}}_{\mathbf{a}(\Delta\varphi,\Delta\omega)\in C^{Lx1}} e^{j\varphi_1}e^{j\omega_1 n}s(n) + \underbrace{\begin{bmatrix} \eta_1(n) \\ \eta_2(n) \\ \cdot \\ \eta_L(n) \end{bmatrix}}_{\boldsymbol{\eta}(n)} \quad (1)$$

where: $\mathbf{x}(n)\in C^{Lx1}, n=1,...,N$ is the vector of the baseband signal samples at the IQ output of receiving channels, N is the number of signal samples in the observation interval T_C, $\mathbf{a}(n)\in C^{Lx1}$ is a column vector whose elements have general form, $\exp(i\Delta\varphi_i)\exp(i\Delta\omega_i), i=1,...,L,\ s(n)$ is the complex envelope of the calibration signal and $\boldsymbol{\eta}(n)\in C^{Lx1}$ is a column vector of noise samples in the receiving channels. Suppose that channel noises are uncorrelated Additive White Gaussian Noise (AWGN) processes with variances equal to σ^2. Unknown parameters of (1) are initial phase and frequency shift $\{\varphi_1,\omega_1\}$ of the signal in the first (referent) channel and relative phase and frequency offsets $\{\Delta\varphi_i,\Delta\omega_i\}, i=1,...,L$ which are to be estimated. The frequency offsets $\{\Delta\omega_i\}$ are normalized as $\Delta\omega_i = 2\pi\Delta f_i / f_s$ where Δf_i is the relative frequency offset and f_s is the sampling frequency. Note that the signal model in (1) is similar to the spatial model of a signal at an antenna array. Vector $\mathbf{a}(n)$ in (1) corresponds to the steering vector in an antenna array signal model. The key difference between these two models is: On one hand there is a strong phase coupling between the elements of a steering vector since the steering vector belongs to the array manifold. On the other hand there is no such coupling in the vector $\mathbf{a}(n)$ in (1) since phase and frequency offsets $\{\Delta\varphi_i,\Delta\omega_i\}, i=1,...,L$ depend on internal PLLs and are generally unpredictable and uncorrelated. However, there is a complete analogy between these models in the case of two receiving channels, and an array processing-type algorithm shall be formulated for estimation of unknown $\{\Delta\varphi,\Delta\omega\}$ parameters. Therefore, our approach is to reduce the problem of estimation of relative phase and frequency offsets in a multichannel receiving system to a successive estimation of relative phase and frequency offsets between the referent and each of the other receiving channels. Suppose that the first channel in a multichannel receiving system is called the referent channel, and the selected ith channel is referred to as the second channel. Mathematical model of such a two-channel receiving system can be expressed as:

$$\begin{bmatrix} x_1(n) \\ x_2(n) \end{bmatrix} = \begin{bmatrix} 1 \\ e^{j\Delta\varphi}e^{j\Delta\omega n} \end{bmatrix} e^{j\varphi_1}e^{j\omega_1 n}s(n) + \begin{bmatrix} \eta_1(n) \\ \eta_2(n) \end{bmatrix} \quad (2)$$

In this case, the estimation problem can be formulated as follows: Let vector $\mathbf{x}(n)\in C^{2x1}, n=1,...,N$ be available, then the relative phase offset $\Delta\varphi$ and the relative frequency offset $\Delta\omega$ between the referent and the second channel shall be estimated. Although in this scenario phase and frequency offsets $\{\varphi_1,\omega_1\}$ of the referent channel are not relevant for array processing applications, from the estimation point of view the uncertainty of $\{\varphi_1,\omega_1\}$ degrades performance of $\{\Delta\varphi,\Delta\omega\}$ estimation.

III. Algorithm for Joint Phase and Frequency Offset Estimation

In literature there are many algorithms for phase/frequency offset estimation such as [4, 5], but they should not be directly applied to solve of the considered calibration problem. As previously stated, unknown parameters $\{\Delta\varphi,\Delta\omega\}$ of the mathematical model (2) are constant in the observation interval so we formulated a Bartlett-type algorithm for joint relative phase and frequency offset estimation $\{\Delta\varphi,\Delta\omega\}$. The following steps are executed for values $\{\Delta\varphi_k,\Delta\omega_l\}$ in the search grid created for the expected range of phase and frequency offsets, $k=1,...N_\varphi,\ l=1,...N_\omega$, where N_φ and N_ω are the numbers of points in the search grid across $\Delta\varphi$ and $\Delta\omega$ dimensions, respectively.

A) For all values $\Delta\omega_l$ calculate:

$$\breve{x}_2(n,\Delta\omega_l) = x_2(n)\exp(-i\Delta\omega_l n); n=1,...,N \quad (3)$$

and form vectors:

$$\breve{\mathbf{x}}(n,\Delta\omega_l) = \begin{bmatrix} x_1(n) \\ \breve{x}_2(n,\Delta\omega_l) \end{bmatrix} \quad (4)$$

B) Calculate covariance matrix:

$$\widehat{\mathbf{R}}(\Delta\omega_l) = 1/N\sum_{n=1}^{N}\breve{\mathbf{x}}(n,\Delta\omega_l)\breve{\mathbf{x}}^H(n,\Delta\omega_l) \quad (5)$$

C) For all values of $\Delta\varphi_k$ form a vector:

$$\mathbf{v}(\Delta\varphi_k) = \begin{bmatrix} 1 & \exp(i\Delta\varphi_k) \end{bmatrix}^T \quad (6)$$

D) Calculate the criterion function:

$$P(\Delta\varphi_k,\Delta\omega_l) = \mathbf{v}(\Delta\varphi_k)^H\widehat{\mathbf{R}}(\Delta\omega_l)\mathbf{v}(\Delta\varphi_k)\ . \quad (7)$$

Criterion function (7) is a Bartlett-like function, well known in array processing [6], but note that vector $\mathbf{v}$ depends only on the phase offset, $\Delta\varphi$ and the covariance matrix depends only on the frequency offset, $\Delta\omega$. Estimated values of the relative phase and frequency offsets $\{\Delta\varphi,\Delta\omega\}$ are determined by finding arguments of the minimum of the function (7), and adding a phase shift of π to the estimated phase. Why is the minimum used for estimation? We consider a two channel system. The minimum of the criterion function appears in the position where signals in two channels are in contra phase but for the appropriate value of the frequency offset. The minimum of the criterion function (7) is much sharper compared to its maximum.

IV. Cramer-Rao Bound (CRB)

Analytic closed form of CRB for the phase and frequency offsets, $\Delta\varphi$ and $\Delta\omega$, estimation for unknown φ_1 and ω_1 are:

$$\mathrm{Var}\Delta\varphi \geq \frac{2(2\mathrm{N}-1)}{\mathrm{N}(\mathrm{N}+1)\mathrm{SNR}}\ ,\quad \mathrm{Var}\Delta\omega \geq \frac{12}{\mathrm{N}(\mathrm{N}^2-1)\mathrm{SNR}} \quad (8)$$

The derivation of these bounds can be found in the Appendix, and it is based on the method described in [7]. Since φ_1 and ω_1 are also unknown parameters, they have to be included in the vector of parameters for estimation in this method, even though their values are irrelevant for the study in this paper.

V. Performance of the Algorithm

Performance of the proposed algorithm is studied by comparing the results of simulations with the derived theoretical CRBs and also illustrated by experimental results using software defined radio (SDR) platforms. The first simulated scenario gives a global qualitative look at the algorithm performance. The values $s(n) = \exp(i\omega_k n);\ n = 0,1,...,N-1,$ are samples (N=100000) of a complex sinusoid with normalized frequency $\omega_k = 0.1$. Frequency offset ω_1 is equal to zero and phase offset φ_1 is randomly generated. Relative phase and frequency offsets in the second channel are: $\Delta\varphi = 151.2609^{\circ}$ and $\Delta\omega = 0.00001$. Noises in the channels are AWGN with unit variances. Signal to noise (SNR) ratio is the same in both channels and equal to 30 dB. Mesh and contour plots of the modulus of the criterion function (7) are shown in Fig. 2(a) and (b), respectively. Estimated values of phase and frequency offset are 151.2842° and 0.00001.

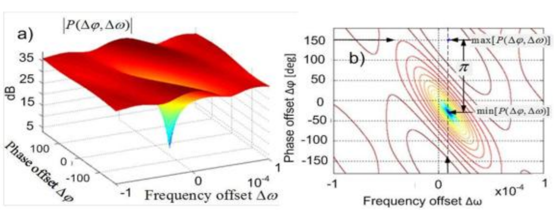

Fig.2. (a) Mesh and (b) contour plot of the modulus of the criterion function (7).

Real and imaginary components (32 samples) of the signals $x_1(n)$ and $x_2(n)$ before and after phase and frequency offset compensation are plotted in Fig. 3(a) and (b), respectively.

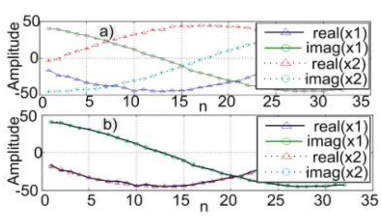

Fig. 3. Signals $x_1(n)$ and $x_2(n)$ (a) before and (b) after phase and frequency offset compensation.

In the second simulation, parameters of the signals were the same, except the value of the frequency offset was $\Delta\omega = 2.5\times10^{-6}$. Results for Mean Square Error (MSE) and corresponding CRBs for phase offset and frequency offset estimation are presented in Figs. 4 and 5, respectively.

The performance of the proposed algorithm is also illustrated by experimental results using two SDR USRP N200 platforms, which are time and frequency synchronized on master-slave principle over MIMO cable. Instantaneous receiver bandwidths were 10 MHz at the central frequencies 900 MHz (real GSM signal). Mesh and contour plots of the modulus of the criterion function (7) are shown in Fig. 6(a) and (b), respectively. Estimated values of phase and frequency offset were 162.8579° and 1.3999×10^{-7}. Many repeated measurements proved that in such a MIMO configuration, phase offsets were randomly distributed in the interval $[-\pi, \pi]$, and frequency offsets were approximately inside the range $\left(-10^{-6}, +10^{-6}\right)$. Number of signal samples used for estimation was $N=10^6$. Real and imaginary components (32 samples) of the signals $x_1(n)$ and $x_2(n)$ before and after phase and frequency offset compensation are plotted in Fig. 7(a) and (b), respectively.

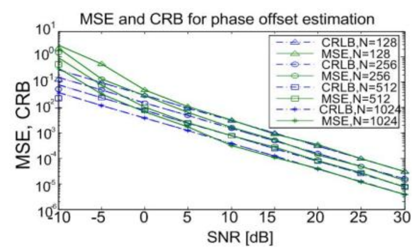

Fig.4. MSE and CRB for phase offset estimation.

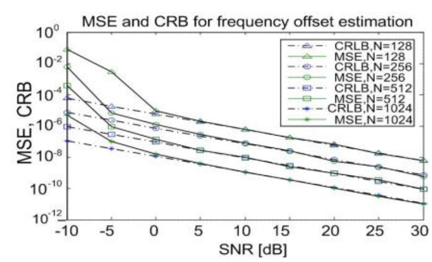

Fig.5. MSE and CRB for frequency offset estimation.

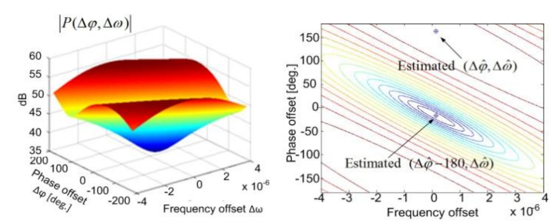

Fig. 6. (a) Mesh and (b) contour plot of the modulus of the criterion function (7) using experimental data.

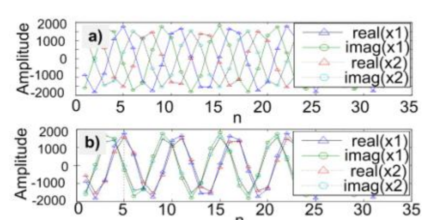

Fig. 7. Signals $x_1(n)$ and $x_2(n)$ a) before and b) after phase and frequency offset compensation in experimental data.

VI. Conclusion

As it can be seen, derived CRB for relative phase and frequency offset decreases as O(1/SNR). Results of simulations presented in Figs. 5 and 6 show that MSE of phase and frequency offset estimation are very close to the corresponding CRB even for low SNR, so the proposed algorithm is statistically efficient. The algorithm does not require a preamble. The authors believe that the proposed procedure could be further improved for calibration of phase/frequency offset in distributed MIMO systems in which a calibration signal generated in a common source should be injected to distributed receivers over the air.

Appendix: The Cramer-Rao Bound for $\{\varphi_1,\omega_1,\Delta\varphi,\Delta\omega\}$ estimation

Let the mathematical model of the received signals in the channels be:

$$x_1(t)=s(t)\,e^{j\varphi_1}e^{j\omega_1 t}+n_1(t)$$

$$x_2(t)=s(t)\,e^{j\varphi_2}e^{j\omega_2 t}+n_2(t)=s(t)\,e^{j\varphi_1}e^{j\omega_1 t}e^{j\Delta\varphi}e^{j\Delta\omega t}+n_2(t)$$

$$\operatorname{Re}\left(n_1(t)\right),\operatorname{Im}\left(n_1(t)\right),\operatorname{Re}\left(n_2(t)\right),\operatorname{Im}\left(n_2(t)\right)\sim N\left(0,\frac{\sigma^2}{2}\right)$$

In this case, the unknown parameters are φ_1, ω_1, $\Delta\varphi$ and $\Delta\omega$. The PDF of $x_1(t)$ is

$$g\left(x_1(k)\,|\,\varphi_1,\omega_1,\Delta\varphi,\Delta\omega\right)=\frac{1}{\pi\sigma^2}e^{-\frac{\left|x_1(k)-s(k)e^{j\varphi_1}e^{j\omega_1 k}\right|^2}{\sigma^2}}$$

and its logarithm is

$$\ln g\left(x_1(k)\,|\,\varphi_1,\omega_1,\Delta\varphi,\Delta\omega\right)=$$
$$-\ln\left(\pi\sigma^2\right)-\frac{1}{\sigma^2}\left|x_1(k)-s(k)e^{j\varphi_1}e^{j\omega_1 k}\right|^2.$$

The logarithm of the PDF of the entire vector $\mathbf{x}$ of samples in both channels is

$$G=\ln g\left(\mathbf{x}\,|\,\varphi_1,\omega_1,\Delta\varphi,\Delta\omega\right)=-2N\ln\left(\pi\sigma^2\right)-$$
$$\frac{1}{\sigma^2}\sum_{k=0}^{N-1}\left(\left|x_1(k)-s(k)e^{j\varphi_1}e^{j\omega_1 k}\right|^2+\left|x_2(k)-s(t)\,e^{j\varphi_1}e^{j\omega_1 k}e^{j\Delta\varphi}e^{j\Delta\omega k}\right|^2\right).$$

The Fisher information matrix (FIM) is

$$I=\left[I_{kl}\right]_{4\times4}=-\mathrm{E}\begin{bmatrix}\frac{\partial^2G}{\partial\varphi_1^2} & \frac{\partial^2G}{\partial\varphi_1\partial\omega_1} & \frac{\partial^2G}{\partial\varphi_1\partial\Delta\varphi} & \frac{\partial^2G}{\partial\varphi_1\partial\Delta\omega}\\ \frac{\partial^2G}{\partial\omega_1\partial\varphi_1} & \frac{\partial^2G}{\partial\omega_1^2} & \frac{\partial^2G}{\partial\omega_1\partial\Delta\varphi} & \frac{\partial^2G}{\partial\omega_1\partial\Delta\omega}\\ \frac{\partial^2G}{\partial\Delta\varphi\partial\varphi_1} & \frac{\partial^2G}{\partial\Delta\varphi\partial\omega_1} & \frac{\partial^2G}{\partial\Delta\varphi^2} & \frac{\partial^2G}{\partial\Delta\varphi\partial\Delta\omega}\\ \frac{\partial^2G}{\partial\Delta\omega\partial\varphi_1} & \frac{\partial^2G}{\partial\Delta\omega\partial\omega_1} & \frac{\partial^2G}{\partial\Delta\omega\partial\Delta\varphi} & \frac{\partial^2G}{\partial\Delta\omega^2}\end{bmatrix}.$$

One of the first order derivatives is

$$\frac{\partial G}{\partial\varphi_1}=-\frac{2}{\sigma^2}\sum_{k=0}^{N-1}\operatorname{Re}\Big(-js(k)e^{j\varphi_1}e^{j\omega_1 k}\left(x_1^*(k)-s^*(k)e^{-j\varphi_1}e^{-j\omega_1 k}\right)-$$
$$js(k)e^{j\varphi_1}e^{j\omega_1 k}e^{j\Delta\varphi}e^{j\Delta\omega k}\Big(x_2^*(k)$$
$$-s^*(k)e^{-j\varphi_1}e^{-j\omega_1 k}e^{-j\Delta\varphi}e^{-j\Delta\omega k}\Big)$$
$$=\frac{2}{\sigma^2}\operatorname{Re}\sum_{k=0}^{N-1}j\Big(x_1^*(k)s(k)e^{j\varphi_1}e^{j\omega_1 k}+$$
$$x_2^*(k)s(k)e^{j\varphi_1}e^{j\omega_1 k}e^{j\Delta\varphi}e^{j\Delta\omega k}-2\left|s(k)\right|^2\Big)$$

The identity

$$\frac{\partial}{\partial\varphi}\left|z(\varphi)\right|^2=\frac{\partial}{\partial\varphi}\left(z(\varphi)z^*(\varphi)\right)=2\operatorname{Re}\left(\frac{\partial z(\varphi)}{\partial\varphi}z^*(\varphi)\right)$$

was used to obtain the previous. One of the second order derivatives is

$$\frac{\partial^2G}{\partial\varphi_1\partial\omega_1}=-\frac{2}{\sigma^2}\operatorname{Re}\sum_{k=0}^{N-1}k\Big(x_1^*(k)s(k)e^{j\varphi_1}e^{j\omega_1 k}+$$
$$x_2^*(k)s(k)e^{j\varphi_1}e^{j\omega_1 k}e^{j\Delta\varphi}e^{j\Delta\omega k}\Big).$$

Some of the elements of the FIM are

$$I_{11}=-\mathrm{E}\frac{\partial^2G}{\partial\varphi_1^2}=\frac{2}{\sigma^2}\operatorname{Re}\sum_{k=0}^{N-1}\Big(\left(\mathrm{E}x_1(k)\right)^*s(k)e^{j\varphi_1}e^{j\omega_1 k}+$$
$$\left(\mathrm{E}x_2(k)\right)^*s(k)e^{j\varphi_1}e^{j\omega_1 k}e^{j\Delta\varphi}e^{j\Delta\omega k}\Big)=$$
$$\frac{2}{\sigma^2}\operatorname{Re}\sum_{k=0}^{N-1}2\left|s(k)\right|^2=\frac{4NP_s}{\sigma^2}=4N\cdot\mathrm{SNR},$$

where $\mathrm{SNR}=\frac{P_s}{\sigma^2}$.

$$I_{12}=I_{21}=-\mathrm{E}\frac{\partial^2G}{\partial\varphi_1\partial\Delta\omega}=\frac{2}{\sigma^2}\operatorname{Re}\sum_{k=0}^{N-1}k\cdot2\left|s(k)\right|^2\approx\frac{4P_s}{\sigma^2}\sum_{k=0}^{N-1}k$$
$$=2N(N-1)\mathrm{SNR}$$

which holds under the reasonable assumption that the energy of the training signal $s(k)$ is approximately evenly distributed in time domain. Finally, the FIM is

$$\mathrm{I}=N\cdot\mathrm{SNR}\begin{bmatrix}4 & 2(N-1) & 2 & N-1\\ 2(N-1) & \frac{2}{3}(N-1)(2N-1) & N-1 & \frac{1}{3}(N-1)(2N-1)\\ 2 & N-1 & 2 & N-1\\ N-1 & \frac{1}{3}(N-1)(2N-1) & N-1 & \frac{1}{3}(N-1)(2N-1)\end{bmatrix}.$$

Therefore, the Cramer-Rao bounds for the phase and frequency offsets, $\Delta\varphi$ and $\Delta\omega$, are

$$\mathrm{Var}\Delta\varphi\geq\left[I^{-1}\right]_{33}=\frac{2(2N-1)}{N(N+1)\mathrm{SNR}}$$

$$\mathrm{Var}\Delta\omega\geq\left[I^{-1}\right]_{44}=\frac{12}{N\left(N^2-1\right)\mathrm{SNR}}$$

where $\left[I^{-1}\right]_{kl}$ is the element of the k-th row and the l-th column of the inverse of the FIM.

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