Outage probability of EGC receiver in TWDP channel

Nenad Milošević, Petar Spalević, Bojan Dimitrijević and Zorica Nikolić

Abstract—**Since the diversity reception is relatively often used in wireless receivers to improve performance in fading channels, this paper investigates equal gain combining (EGC) diversity receiver performance in a fading channel. More precisely, the outage probability of the EGC receiver in two wave with diffuse power (TWDP) fading channel will be analyzed. The influence of different fading parameters on the system performance will be considered through the theoretical analysis and simulation. Finally, some topics for the future work in this area will be proposed.**

Index Terms—**Outage probability; Equal gain combining; Two wave with diffuse power fading.**

I. Introduction

IN wireless communications, the transmitted signal reflects from different obstacles and the receiver receives multiple copies of the transmitted signal with different delays. This multipath propagation causes the variation of the instantaneous value of the received signal, or fading. Fading is one of the main causes of performance degradation in wireless communication systems. The fading channels are usually described by Rayleigh, Rician or Nakagami-m probability density functions (pdfs) [1]. However, certain fading situations cannot be characterized by the fading models mentioned above. A newer, more general fading model - two wave with diffuse power (TWDP) was introduced and investigated in [2]. For TWDP fading, there are two specular multipath components in the presence of diffusely propagating waves. A research [3] demonstrated that TWDP fading channel may produce worse propagation conditions than Rayleigh fading channel if the two direct waves are equal in strength and have a combined power of more than 6 dB higher than the diffuse power. This is an important result since the Rayleigh fading is often considered as the worst-case scenario in the design of communication links. Also, by choosing the TWDP fading parameters, it may be simplified to Rayleigh or Rician fading.

Nenad Milošević is with the Faculty of Electronic Engineering, University of Niš, 14 Aleksandra Medvedeva, 18000 Niš, Serbia (e-mail: nenad.milosevic@elfak.ni.ac.rs).

Petar Spalević is with the Faculty of Technical Sciences, University of Pristina, st. Knjaza Milosa 7, K. Mitrovica, Serbia (e-mail: petarspalevic@yahoo.com).

Bojan Dimitrijević is with the Faculty of Electronic Engineering, University of Niš, 14 Aleksandra Medvedeva, 18000 Niš, Serbia (e-mail: bojan.dimitrijevic@elfak.ni.ac.rs).

Zorica Nikolić is with the Faculty of Electronic Engineering, University of Niš, 14 Aleksandra Medvedeva, 18000 Niš, Serbia (e-mail: zorica.nikolic@elfak.ni.ac.rs).

Since fading is always present in wireless channels, some techniques should be used to combat the harmful effects of it. A relatively simple and often used techniques is the diversity reception. It is particularly significant since it does not require higher transmitting power or bandwidth. Diversity receiver receives independent multiple copies, usually using spatially separated receiver antennas, of the transmitted signal through different fading paths. These copies are combined and the signal to noise ratio (SNR) is improved. The diversity receiver is able to improve SNR because it is not very likely that all fading paths go into a deep fade at the same time. Particular diversity methods and combining techniques are presented in [1], [4]. Diversity reception in TWDP channel is studied in [5 – 12]. The optimum diversity technique, maximal ratio combining (MRC) is analyzed in [5 – 7]. Bit error rate (BER) for binary phase shift keying (BPSK) modulation is considered in [5], and quadrature amplitude modulation (QAM) BER is studied in [6]. Reference [7] presented symbol error rate (SER) analysis of the coherent and non-coherent m-ary modulation techniques using M-MRC diversity receiver. Selection combining (SC), the simplest combining technique, is considered in [8 – 11]. These papers analyze BER, SER [8], [9], the outage probability [9], [10], and the system capacity [11]. The third combining technique, often used in telecommunication systems, is the equal gain combining (EGC). The performance of the EGC receiver is very close to the performance of the MRC, with an advantage that the complexity is considerably lower [1]. However, EGC is not often analyzed within the receivers operating in TWDP channels. BER of the postdetection EGC for noncoherent frequency shift keying (NCFSK) and differential phase shift keying (DPSK) is considered in [12]. Reference [13] analyzes dual-EGC (EGC with two diversity branches) receiver in TWDP channel.

In this paper we extend the analysis from [13] and consider *M*-EGC (EGC with *M* diversity branches). Theoretical and simulation results for the outage probability will be given for different channel conditions.

The rest of the paper is organized as follows. Section II describes TWDP fading model. The outage probability is derived in Section III, and the numerical results are given in Section IV. The conclusion summarizes the results and provides proposal for the future work.

II. Channel and System Model

Although there is no an exact closed form of amplitude probability density function (pdf) for TWDP fading, in [2] authors proposed a family of approximate pdfs. The approximation of TWDP channel model includes two parameters $K=(V_1^2+V_2^2)/2\sigma^2$ and $\Delta=2V_1V_2/(V_1^2+V_2^2)$, where V_1 and V_2 are voltage magnitudes of two specular waves, and $2\sigma^2$ is the average power of the diffuse waves. K represents the ratio of total specular power to diffuse power, and Δ describes the relative strength of the two specular waves. For different combinations of K and Δ, the fading channel behaves differently, as shown in Table I [3].

TABLE I
TWDP FADING CHANNEL BEHAVIOR

Parameter(s) Value	Behavior
$K \le 2$	~ Rician fading
$K\Delta \le 2$	~ Rician fading
$K=0$	~ Rayleigh fading
$K<\min\left\{2/\Delta, 1/\sqrt{(1-\Delta^2)-1}\right\}$	~ Rayleigh fading
As $K\to 10, \Delta\to 1$	Deviate from Rician
Large K, $\Delta\approx 1$	BER poorer than Rayleigh

The channel is assumed to be slow, frequency nonselective, with TWDP fading statistics. The complex low pass equivalent of the received signal over one symbol duration T_s can be expressed as

$$r'(t)=re^{j\phi}s(t)+n(t)\ , \qquad (1)$$

where $s(t)$ is the transmitted symbol, E_s is transmitted symbol energy and $n(t)$ is the complex Gaussian noise having zero mean and two-sided power spectral density $2N_0$. Random variable ϕ represents the phase and r is the TWDP distributed fading amplitude. Without the loss of generality, we will assume that the channel coefficients in all branches are identically distributed random variables. Therefore, the received signal amplitude pdf in each branch is equal to [2]

$$f_{r_m}(r)=\frac{r}{\sigma^2}\exp\left(\frac{-r^2}{2\sigma^2}-K\right)\sum_{i=1}^{L}a_iD\left(\frac{r}{\sigma};K,\alpha_i\right), \qquad (2)$$
$$m=1,2,\ldots$$

where

$$\alpha_i=\Delta\cos\frac{\pi(i-1)}{2M-1}$$
$$D(x;K,\alpha)=\frac{1}{2}\exp(\alpha K)I_0\left(x\sqrt{2K(1-\alpha)}\right)+ \qquad (3)$$
$$+\frac{1}{2}\exp(-\alpha K)I_0\left(x\sqrt{2K(1+\alpha)}\right)$$

Values for a_i, for L = 1…5 are given in Table II [2]:

TABLE II
COEFFICIENTS OF *a* FOR THE TWDP PDF

L	a_1	a_2	a_3	a_4	a_5
1	1				
2	$\frac{1}{4}$	$\frac{3}{4}$			
3	$\frac{19}{144}$	$\frac{25}{48}$	$\frac{25}{72}$		
4	$\frac{75}{8640}$	$\frac{3577}{8640}$	$\frac{49}{320}$	$\frac{2989}{18640}$	
5	$\frac{2857}{44800}$	$\frac{15741}{44800}$	$\frac{27}{1120}$	$\frac{1209}{2800}$	$\frac{2889}{22400}$

The SNR at the output of the EGC receiver is given by [14]:

$$\gamma_{\text{out}}=\frac{E_s}{N_0}\frac{1}{M}\left(\sum_{m=1}^{M}r_m\right)^2, \qquad (4)$$

where r_m is the received signal envelope in mth input path, modeled as TWDP random variable.

III. PERFORMANCE ANALYSIS

In order to determine the outage probability, we need first to determine the statistics of the instantaneous signal to noise ratio. The characteristic function of the received signal envelope in mth input path is, by definition:

$$\varphi_{r_m}(\omega)=E\left\langle e^{j\omega r}\right\rangle=\int_0^\infty f_{r_m}(r)e^{j\omega r}dr\ . \qquad (5)$$

The characteristic function of the sum of the received signal envelopes

$$S=\sum_{m=1}^{M}r_m \qquad (6)$$

is equal to

$$\varphi_S(\omega)=\varphi_{r_1}(\omega)\times\varphi_{r_2}(\omega)\times\cdots\ )\ . \qquad (7)$$

Since random variables in all branches are identically distributed, (7) becomes

$$\varphi_S(\omega)=(\varphi_{r_1}(\omega))^M\ . \qquad (8)$$

The pdf for the random variable S may be obtained by taking inverse Fourier transform of (8):

$$p_S(r)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\varphi_S(\omega)e^{-j\omega r}d\omega\ . \qquad (9)$$

Now, pdf of the squared sum S^2 is

$$p_{S^2}(r) = \frac{1}{2\sqrt{r}} p_S\left(\sqrt{r}\right). \quad (10)$$

Finally, pdf of the EGC output SNR is

$$f_\gamma(\gamma) = \frac{M}{E_s/N_0} f_{S^2}\left(\gamma \frac{M}{E_s/N_0}\right). \quad (11)$$

The outage probability is defined as a probability that the output SNR drops below a certain threshold γ_{th} :

$$P_{out} = \Pr\left(\gamma < \gamma_{\text{th}}\right) = \int_0^{\gamma_{\text{th}}} f_\gamma(\gamma) d\gamma . \quad (12)$$

IV. Numerical Results

Since there is no closed form for the expressions (7) to (12), the results are obtained by numerical integration. Also, the Monte-Carlo simulation results prove the validity of the theoretical expressions. For all figures, the outage probability threshold is $\gamma_{\text{th}} = 5$ dB.

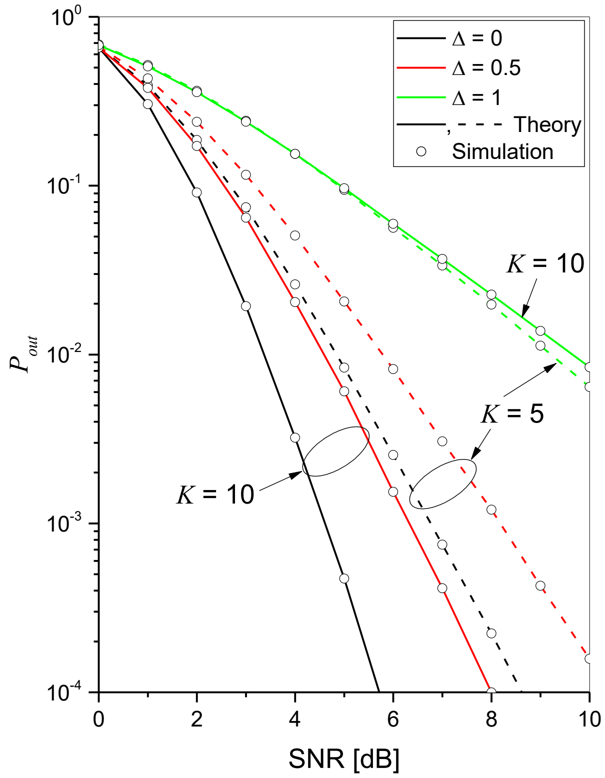

Fig. 1. Outage probability as a function of SNR for different fading parameters and $M = 3$.

Fig. 1. depicts the outage probability as a function signal to noise ratio, for different fading parameters and $M = 3$ diversity branches in the receiver. As can be seen, for higher Δ the outage probability is higher. Also, for higher *K*, the total power of specular waves is higher compared to the power of diffuse waves, and the outage probability is lower.

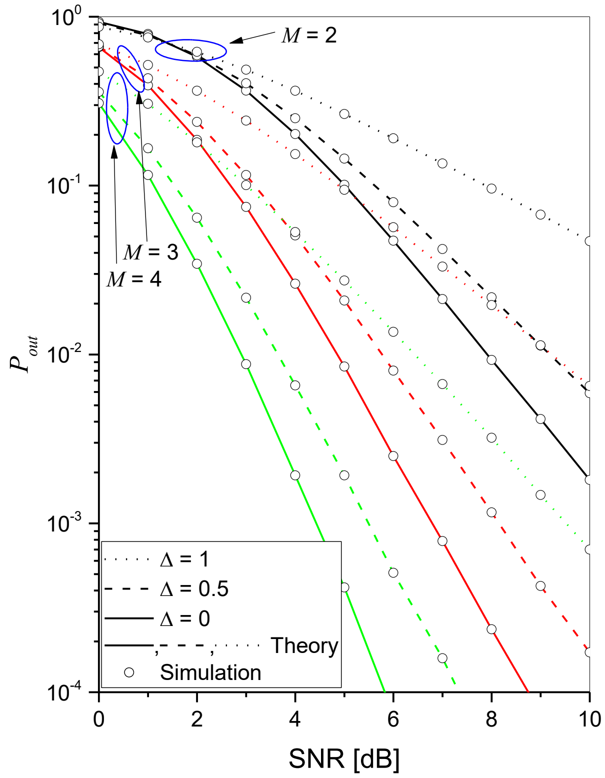

Fig. 2. Outage probability as a function of SNR for different number of receiver branches.

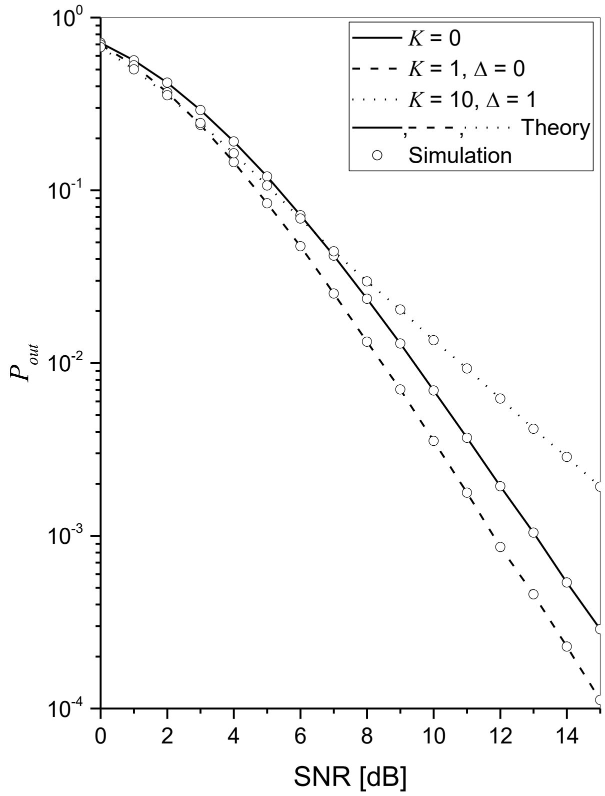

Fig. 3. Outage probability as a function of SNR for different fading severity and $M = 3$.

The influence of the number of the diversity branches in the receiver is shown in Fig. 2. As expected, the higher the number of branches, the lower outage probability is.

Finally, the outage probability as a function of SNR for different fading severity is shown in Fig. 3, for $M = 3$ diversity branches in the receiver. There are three combinations of parameters: ($K = 1$, $\Delta = 0$, one direct wave with diffuse component), which models Rician fading conditions, ($K = 0$, $\Delta = 0$, diffuse component without direct wave), which represents Rayleigh fading, and ($K = 10$, $\Delta = 1$), which creates fading worse than Rayleigh fading. It may be noticed that the conditions worse than Rayleigh appear for SNR > 7 dB, and the difference is higher for higher SNR.

All three figures demonstrate an excellent match between theoretical and simulation curves.

V. Conclusion

The outage probability for M branch EGC receiver in TWDP fading channel is analyzed. The theoretical expressions are derived and the performance of the considered system are demonstrated, both theoretical and simulation. The results show the importance of TWDP fading model, since it is able to create communication channel worse than Rayleigh.

As a future work, it may be significant to explore the performance of the cooperative relaying systems in TWDP channel.

Acknowledgment

The research leading to these results has received funding from the Ministry of Education, Science and Technological Development of Serbia within the projects "Development and implementation of next-generation systems, devices and software based on software radio for radio and radar networks" (TR-32051), and “Research and development of robust transmission systems for corporative networks” (TR-32037).

References

[1] M. K. Simon, M-S. Alouini, *Digital Communication over Fading Channels*, 2nd ed. New Jersey, USA: John Wiley & Sons. 2005.

[2] G. D. Durgin, T. S. Rappaport, D. A. de Wolf, “New analytical models and probability density functions for fading in wireless communications,” *IEEE Trans. Commun.*, vol. 50, no. 6, pp. 1005-1015, Jun 2002.

[3] S. H. Oh, K. H. Li, “BER performance of BPSK receivers over two-wave with diffuse power fading channels,” *IEEE Trans. Wireless Commun.*, vol. 4, no. 4, pp. 1448-1454, July 2005.

[4] J. G. Proakis, *Digital Communications*. 2nd ed. USA, New York: McGraw-Hill. 2000.

[5] S. H. Oh, K. H. Li, W. S. Lee, “Performance of BPSK predetection MRC systems over two-wave with diffuse power fading channels,” *IEEE Trans. Wirel. Commun.*, vol 6, no. 8, pp. 2772–2775, Aug. 2007.

[6] Y. Lu, N. Yang, “Symbol error probability of QAM with MRC diversity in two-wave with diffuse power fading channels,” *IEEE Commun. Letters*, vol. 15, no. 1, pp. 10-12, Jan. 2011.

[7] R. Subadar, A. D. Singh, “Performance of M-MRC systems over TWDP fading channels,” *Intern. J. Electr. and Commun. (AEU)*, vol. 68, no. 6, pp. 569–572, Jun 2014.

[8] B. S. Tan, K. H. Li, K. C. Teh, “Symbol error rate of selection combining over two wave with diffuse power fading,” Proc. International Symposium on Intelligent Signal Processing and Communications Systems (ISPACS 2011), Chiang Mai, Thailand, pp. 1-5, 07 Dec - 09 Dec, 2011.

[9] R. Subadar, A. D. Singh, “Performance of SC receiver over TWDP fading channels,” *IEEE Wireless Commun. Letters*, vol. 2, no. 3, pp. 267-270, Jun. 2013.

[10] N. Milošević, B. Dimitrijević, Z. Nikolić, “Performance analysis of selection and switch-and-stay combining diversity receivers in two wave with diffuse power fading channel,” Proc. PES 2013, Niš, Serbia, 01 Sep – 04 Sep, 2013.

[11] A. D. Singh, R. Subadar, “Capacity analysis of M-SC receivers over TWDP fading channels,” *Intern. J. Electr. and Commun. (AEU)*, vol. 68, no. 2, pp. 166-171, Feb. 2014.

[12] W. S. Lee, “Performance of postdetection EGC NCFSK and DPSK systems over two-wave with diffuse power fading channels,” Proc. International Symposium on Communications and Information Technologies (ISCIT’07), Sydney, Australia, pp. 911-915, 17 Oct – 19 Oct 2007.

[13] A. D. Singh, R Subadar, “Performance of Dual-EGC receivers over TWDP fading channels,” *Telecommun. Systems*, vol. 63, no. 3, pp. 473-479, Nov. 2016.

[14] D. A. Zogas, G. K. Karagiannidis, S. A. Kotsopoulos, “Equal gain combining over Nakagami-n (Rice) and Nakagami-q (Hoyt) generalized fading channels,” *IEEE Trans. Wireless Commun.*, vol. 4, no. 2, pp. 374-379, Mar. 2005.