Average BER of Dual-Branch FSO System Employing SIM-BPSK Influenced by Malaga Atmospheric Turbulence with Pointing Errors

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***Abstract*— This paper presents an average bit error rate (BER) analysis of the free-space optical (FSO) system employing subcarrier intensity modulation (SIM) with binary phase-shift keying (BPSK). Intensity fluctuations due to atmospheric turbulence are modeled by Malaga (M) distribution. In addition, pointing errors effect is taken into account. Novel closed-form average BER expression for SIM-BPSK FSO system is derived. Furthermore, dual-branch FSO system is considered, when equal gain combining (EGC) is employed, and average BER expression is derived. Numerical results are presented and validated by Monte Carlo simulations. The effects of atmospheric turbulence and pointing errors parameters on the average BER performance are discussed. It is concluded that employing EGC at the receiver results in the FSO system performance improvement, especially when the channel conditions are favorable.**

***Index Terms*— Atmospheric turbulence, binary phase-shift keying, bit error rate, equal gain combining, free-space optics, Malaga distribution, pointing errors.**

I. INTRODUCTION

THE free-space optical (FSO) system represents modern attractive technology which has gathered attention of research community in the past decade. The FSO systems are characterized by wide bandwidth and data rates similar as the ones provided by the optical fiber systems, as well as easy, low-cost and simple installation. Also, the FSO is license-free transmission in unregulated spectrum, performed in outdoor environment [1] − [3].

The main reason for the intensity fluctuations of received optical signal is the existence of the atmospheric turbulence. This phenomenon occurs due to variations in atmospheric temperature and pressure, which lead to the random changes in the refractive index. Although many statistical models were adopted to describe the effect of atmospheric turbulence, recently proposed Malaga (M) distribution has received an attention in scientific society since it represents general model and can be reduced to well-known distributions (Log-normal, K, Gamma-Gamma, Exponential, etc.). Generality of the M distribution lies in the fact that the effect of multiple scattered components is taken into consideration [4] − [9]. Another cause of the intensity fluctuations of received optical signal and system performance degradation is the misalignment between transmitter laser and receiver photodetector. Due to many phenomena, such as earthquakes, strong wind and thermal expansion, buildings (together with installed FSO apertures) are moving, which cause optical laser beam vibrations. Hence, the positioning of the laser beam will not be precise, and the misalignment between transmit laser and receiver photodetector occurs, which represents the effect called pointing errors [10] − [12]. The statistical model which takes into account both M-distributed atmospheric turbulence and pointing errors was analyzed in [7] − [9].

In order to mitigate effect of atmospheric turbulence and pointing errors and to improve system performance, many methods have been borrowed from traditional radio-frequency systems. As one of widely used, spatial diversity techniques were investigated to be employed within FSO systems [1], [2]. The FSO signal transmission via K-distributed atmospheric turbulence channel, when selection combining (SC), equal gain combining (EGC) or maximum ratio combining (MRC) is employed, was analyzed in [13]. The analysis of the FSO system influenced by weak log-normal atmospheric turbulence conditions, employing SC and switch-and-stay combining (SSC)/switch-and-examine (SEC) combining, was presented in [14] and [15], respectively. Assuming Gamma–Gamma atmospheric turbulence conditions, implementation of the spatial diversity techniques was observed in [16], [17], while the effect of the pointing errors was added in [18] − [20]. The FSO system affected by atmospheric turbulence modeled by M-distribution employing diversity techniques was analyzed in [21], [22]. More precisely, the average bit error rate (BER) analysis of the dual-branch FSO system with MRC and EGC was presented in [21]. In [22], the average BER of the FSO system with multiple receivers, employing MRC and SC, was derived, assuming that pointing errors effect is taken into account.

In this paper, the FSO system with subcarrier intensity modulation (SIM) employing binary phase-shift keying (BPSK), considering M-distributed atmospheric turbulence and pointing errors effect, is analyzed. The closed-form average BER expression is derived for the single-input single-output (SISO) system. Furthermore, the average BER expressions for the dual-branch FSO system are derived when EGC diversity techniques is employed at the receiver. Based

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on derived expressions, numerical results are presented and validated by Monte Carlo simulations.

The rest of the paper is organized as follows. The system and channel model is given in Section II. In Section III, the average BER analysis is presented for both SISO and dual-branch FSO systems. Numerical results with corresponding comments are given in Section IV. Concluding remarks are presented in Section V.

II. SYSTEM AND CHANNEL MODEL

Dual-branch FSO system with EGC diversity techniques applied at the receiver is investigated. The SIM with BPSK scheme is considered at the transmitting part of the system. It means that the information bits are first premodulated by BPSK modulator. The signal at the output of modulator is used to modulate the laser source intensity. DC bias is added in order to satisfy non-negativity requirement. Finally, the laser transmits optical beam via atmospheric channel. Dual-branch system is considered, i.e., there are two receiver photodetectors to collect optical beam. The independent and identically distributed (i.i.d.) branches are considered. The optical signal at the i-th receiving aperture is defined as

$$s_i = P_t I_i \left(1 + m\cos\left(2\pi f_c t + x_k \pi\right)\right),\ \ i = 1,2, \tag{1}$$

where P_t is the average transmitted optical power, I_i is normalized signal irradiance of the i-th branch, m represents modulation index (neglected since m=1), and f_c is the subcarrier frequency. The binary information data is denoted by $x_k \in \{0,1\}$. DC bias is removed and optical-to-electrical conversion is performed by PIN photodetectors. The electrical signal at the i-th receiving aperture is defined as

$$r_i = RP_t I_i \cos\left(2\pi f_c t + x_k \pi\right) + n_i,\ \ i = 1,2, \tag{2}$$

where R is each detector responsivity, and n_i is the additive white Gaussian noise with zero mean and variance σ_{ni}^2.

The received instantaneous SNR, γ_i, for each branch is defined as

$$\gamma_i = \frac{R^2 I_i^2 P_t^2}{2\sigma_{ni}^2},\ \ i = 1,2, \tag{3}$$

while the average electrical SNR, μ_i, for each branch can be determined as

$$\mu_i = \frac{R^2 P_t^2}{2\sigma_{ni}^2} \mathrm{E}^2\left[I_i\right],\ \ i = 1,2, \tag{4}$$

where E[.] is the statistical expectation.

The statistic of the signal intensity, I_i, is accounted for M-distributed atmospheric turbulence and pointing errors. The M distribution is more general model compared to already known ones, since it takes into account the effect of multiple scattered components. As in the case of Gamma-Gamma, besides the component $\mathrm{C_L}$ which occurs due to LOS contribution, the component $\mathrm{C_{CS}}$ scattered by the eddies on the propagation axis is also considered. This component is coupled to $\mathrm{C_L}$. In addition, M statistical model takes into account the component scattered to the receiver by the off-axis eddies, $\mathrm{C_{GS}}$. This component is statistically independent from both $\mathrm{C_L}$ and $\mathrm{C_{CS}}$ [4]. Taking into account M-distributed atmospheric turbulence and pointing errors, the probability density function (PDF) of I_i is given by [7, (21)]

$$\begin{aligned} f_{I_i}(I) = {} & \frac{\xi^2 \mathrm{A}}{2I} \sum_{k=1}^{\beta} a_k \left(\frac{\alpha\beta}{g\beta + \Omega'}\right)^{-\frac{\alpha+k}{2}} \\ & \times G_{1,3}^{3,0}\left(\frac{\alpha\beta}{g\beta + \Omega'} \frac{I}{\mathrm{A}_0} \middle| \begin{matrix} \xi^2 + 1 \\ \xi^2,\ \ \alpha,\ \ k \end{matrix}\right), \end{aligned} \tag{5}$$

where $G_{p,q}^{m,n}(\cdot)$ denotes the Meijer's G-function [23, (9.301)], and the constants are defined as [7, (8)]

$$\begin{aligned} \mathrm{A} & \triangleq \frac{2\alpha^{\frac{\alpha}{2}}}{g^{1+\frac{\alpha}{2}}\Gamma(\alpha)} \left(\frac{g\beta}{g\beta + \Omega'}\right)^{\beta + \frac{\alpha}{2}}, \\ a_k & \triangleq \binom{\beta - 1}{k - 1} \frac{\left(g\beta + \Omega'\right)^{1-\frac{k}{2}}}{(k-1)!} \left(\frac{\Omega'}{g}\right)^{k-1} \left(\frac{\alpha}{\beta}\right)^{\frac{k}{2}}, \end{aligned} \tag{6}$$

with β represents a natural number representing the amount of fading parameter and α is a positive parameter related to the effective number of large-scale cells of the scattering process. The average power of the scattering component received by off-axis eddies is defined as

$$g = \mathrm{E}\left[\left|\mathrm{C_{GS}}\right|^2\right] = 2b_0\left(1 - \rho_M\right), \tag{7}$$

where $2b_0 = \mathrm{E}\left[\left|\mathrm{C_{CS}}\right|^2 + \left|\mathrm{C_{GS}}\right|^2\right]$ represents the average power of the total scatter components. Parameter ρ_M defines the amount of scattering power coupled to the LOS component $\left(0 \le \rho_M \le 1\right)$. The parameter $\Omega = \mathrm{E}\left[\left|\mathrm{C_L}\right|^2\right]$ is the average power of the LOS component is, while $\Omega' = \Omega + 2b_0\rho_M + 2\sqrt{2b_0\rho_M\Omega}\cos\left(\phi_A - \phi_B\right)$ is the average power from the coherent contributions, with deterministic phases of the LOS and the coupled-to-LOS scatter terms ϕ_A and ϕ_B, respectively [4] – [9].

The pointing errors parameter ξ is defined as the ratio between equivalent beam radius at receiver, denoted as a_{dqe}, and jitter (pointing error) standard deviation, denoted as σ_s, as

$$\xi = \frac{a_{deq}}{2\sigma_s}, \tag{8}$$

The parameter a_{deq} is related to the optical beam radius at the distance d, a_d, as $a_{deq}^2 = a_d^2 \sqrt{\pi}\operatorname{erf}(v) \big/ \left(2v\exp\left(-v^2\right)\right)$, where $v = \sqrt{\pi}a \big/ \left(\sqrt{2}a_d\right)$ and a is the radius of a circular detector aperture [10] − [12]. The parameter $A_0 = \operatorname{erf}^2(v)$ represents the maximal fraction of the collected power, where erf(.) is the error function [23, Eq. (8.250.1)]. The Rytov variance is used for determination of the optical signal intensity due to atmospheric turbulence. It is defined as $\sigma_R^2 = 1.23 C_n^2 \left(\frac{2\pi}{\lambda}\right)^{7/6} d^{11/6}$, with the wavelength λ, propgation distance d, and the refractive index structure parameter C_n^2.

Based on instantaneous SNR in (3), after mathematical manipulations, the PDF of γ_i is derived as [8, (7)]

$$\begin{aligned} f_{\gamma_i}(\gamma) = {} & \frac{\xi^2 \mathrm{A}}{4\gamma} \sum_{k=1}^{\beta} a_k \left(\frac{\alpha\beta}{g\beta + \Omega'}\right)^{-\frac{\alpha+k}{2}} \\ & \times G_{1,3}^{3,0}\left(\frac{\alpha\beta\kappa(g+\Omega')}{g\beta+\Omega'}\sqrt{\frac{\gamma}{\mu_i}}\,\middle|\, \begin{matrix} \xi^2+1 \\ \xi^2,\ \alpha,\ k \end{matrix}\right), \end{aligned} \tag{9}$$

where $\kappa = \xi^2 \big/ \left(\xi^2+1\right)$. Based on the definition in (4), and the PDF of signal intensity, I_i, in (5), and the average electrical SNR is determined as

$$\mu_i = \frac{\left(RP_t\right)^2}{2\sigma_{ni}^2} A_0^2 \kappa^2 \left(g+\Omega'\right)^2, \ i = 1,2. \tag{10}$$

III. AVERAGE BER ANALYSIS

In the following Section, the average BER analysis is presented. Firstly, the novel closed-form expression for the SISO FSO system employing SIM-BPSK is derived. Further, the average BER expression for dual-branch SIM-BPSK FSO system with EGC diversity technique is obtained.

A. SISO FSO system

The conditional average BER of the SISO FSO system with SIM-BPSK is expressed as [24]

$$P_{b/\gamma} = \frac{1}{2}\operatorname{erfc}\left(\sqrt{\gamma}\right), \tag{11}$$

where erfc(.) is the complementary error function [23, (8.250.4)], and γ is the previously defined instantaneous electrical SNR in (3). Index i is omitted since SISO channel with only one receiver detector is considered $\left(f_{\gamma_i}(\gamma) = f_\gamma(\gamma)\right)$. The average BER of SISO FSO system is obtained by averaging (11) over γ as

$$P_b = \frac{1}{2}\int_0^\infty \operatorname{erfc}\left(\sqrt{\gamma}\right) f_\gamma(\gamma)\, d\gamma. \tag{12}$$

After substituting (9) into (12), the BER is expressed as

$$\begin{aligned} P_b = {} & \frac{\xi^2 \mathrm{A}}{8} \sum_{k=1}^{\beta} a_k \left(\frac{\alpha\beta}{g\beta+\Omega'}\right)^{-\frac{\alpha+k}{2}} \int_0^\infty \gamma^{-1} \operatorname{erfc}\left(\sqrt{\gamma}\right) \\ & \times G_{1,3}^{3,0}\left(\frac{\alpha\beta\kappa(g+\Omega')}{g\beta+\Omega'}\sqrt{\frac{\gamma}{\mu}}\,\middle|\, \begin{matrix} \xi^2+1 \\ \xi^2,\ \alpha,\ k \end{matrix}\right) d\gamma. \end{aligned} \tag{13}$$

The first step in order to solve integral in (13) is transformation of the complementary error function in terms of Meijer's *G*-function by using [25, (06.27.26.0006.01)] as

$$\operatorname{erfc}\left(\sqrt{\gamma}\right) = \frac{1}{\sqrt{\pi}} G_{1,2}^{2,0}\left(\gamma \,\middle|\, \begin{matrix} 1 \\ 0,\ 1/2 \end{matrix}\right) \tag{14}$$

After substituting (14) into (13), the average BER is written as

$$\begin{aligned} P_b = {} & \frac{\xi^2 \mathrm{A}}{8\sqrt{\pi}} \sum_{k=1}^{\beta} a_k \left(\frac{\alpha\beta}{g\beta+\Omega'}\right)^{-\frac{\alpha+k}{2}} \int_0^\infty \gamma^{-1} G_{1,2}^{2,0}\left(\gamma \,\middle|\, \begin{matrix} 1 \\ 0,\ 1/2 \end{matrix}\right) \\ & \times G_{1,3}^{3,0}\left(\frac{\alpha\beta\kappa(g+\Omega')}{g\beta+\Omega'}\sqrt{\frac{\gamma}{\mu}}\,\middle|\, \begin{matrix} \xi^2+1 \\ \xi^2,\ \alpha,\ k \end{matrix}\right) d\gamma. \end{aligned} \tag{15}$$

Integral in (15) is solved based on [25, (07.34.21.0013.01)] as

$$\begin{aligned} P_b = {} & \sum_{k=1}^{\beta} \frac{2^{\alpha+k-5}\xi^2 \mathrm{A} a_k}{\pi^{3/2}} \left(\frac{\alpha\beta}{g\beta+\Omega'}\right)^{-\frac{\alpha+k}{2}} \\ & \times G_{4,7}^{6,2}\left(\frac{\alpha^2\beta^2\kappa^2\left(g+\Omega'\right)^2}{16\mu\left(g\beta+\Omega'\right)^2} \,\middle|\, \begin{matrix} 1, \frac{1}{2}, \frac{\xi^2+1}{2}, \frac{\xi^2+2}{2} \\ \frac{\xi^2}{2}, \frac{\xi^2+1}{2}, \frac{\alpha}{2}, \frac{\alpha+1}{2}, \frac{k}{2}, \frac{k+1}{2}, 0 \end{matrix}\right). \end{aligned} \tag{16}$$

The order of Meijer's *G*-function in (16) can be reduced by [23, (07.34.03.0002.01)]. Afterwards, the closed-form expression for SISO SIM-BPSK FSO system is

$$\begin{aligned} P_b = {} & \sum_{k=1}^{\beta} \frac{2^{\alpha+k-5}\xi^2 \mathrm{A} a_k}{\pi^{3/2}} \left(\frac{\alpha\beta}{g\beta+\Omega'}\right)^{-\frac{\alpha+k}{2}} \\ & \times G_{3,6}^{5,2}\left(\frac{\alpha^2\beta^2\kappa^2\left(g+\Omega'\right)^2}{16\mu\left(g\beta+\Omega'\right)^2} \,\middle|\, \begin{matrix} 1, \frac{1}{2}, \frac{\xi^2+2}{2} \\ \frac{\xi^2+1}{2}, \frac{\alpha}{2}, \frac{\alpha+1}{2}, \frac{k}{2}, \frac{k+1}{2}, 0 \end{matrix}\right). \end{aligned} \tag{17}$$

When $\rho_M = 1$, it holds that the average power of the scattering component received by off-axis eddies, earlier defined as *g*, is equal to zero. It means that there are no the component C_{GS}, and M distribution is reduced to the Gamma-Gamma distribution. In that case, the product of constants A and a_k is nonzero only when $k = \beta$. Hence, the average BER expression is obtained as

$$P_b^{GG} = \frac{2^{\alpha+\beta-4}\xi^2}{\pi^{3/2}\Gamma(\alpha)\Gamma(\beta)}\left(g\beta+\Omega'\right)^{\frac{\alpha+\beta}{2}} \times G_{3,6}^{5,2}\left(\frac{\alpha^2\beta^2\kappa^2\left(g+\Omega'\right)^2}{16\mu\left(g\beta+\Omega'\right)^2}\left|\begin{matrix} 1,\frac{1}{2},\frac{\xi^2+2}{2} \\ \frac{\xi^2+1}{2},\frac{\alpha}{2},\frac{\alpha+1}{2},\frac{k}{2},\frac{k+1}{2},0 \end{matrix}\right.\right). \quad (18)$$

B. Dual-branch FSO system employing EGC

Assuming that EGC diversity technique with two apertures is employed at the reception, received photocurrents from each FSO aperture are coherently summed with equal weights of unity. The PDF of the irradiance at the EGC output, I_{EGC}, is the joint PDF of the vector $\mathbf{I} = (I_1, I_2)$, determined as

$$f_{I_{EGC}}(I) = \int_0^I f_{I_1}(I - I_2) f_{I_2}(I_2) dI_2. \quad (19)$$

In this case, the average BER can be found by averaging conditional average BER in (11) over γ_{EGC}. Otherwise, after replacing (3) into (11), the conditional average BER as re-written as

$$P_{b/I} = \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{R^2 I^2 P_t^2}{2\sigma_{ni}^2}}\right). \quad (20)$$

The average BER can be determined by averaging conditional average BER in (20) over irradiance at EGC output, I_{EGC}, as

$$P_b^{EGC} = \frac{1}{2}\int_0^\infty \operatorname{erfc}\left(\sqrt{\frac{R^2 I^2 P_t^2}{2\sigma_{ni}^2}}\right) f_{I_{EGC}}(I) dI, \quad (21)$$

where the PDF of the irradiance at the EGC output, I_{EGC}, is given in (19). If we want to express the average BER expression in terms of average electrical SNR, correspond alternation is performed based on (10). The average BER is obtained as

$$P_b^{EGC} = \frac{1}{2}\int_0^\infty \operatorname{erfc}\left(\sqrt{\frac{\mu_i}{A_0^2\kappa^2\left(g+\Omega'\right)^2} I^2}\right) f_{I_{EGC}}(I) dI. \quad (22)$$

Integral in (22) has no closed form, and the final BER expression is evaluated numerically.

IV. NUMERICAL RESULTS

In this Section, numerical results are obtained based on derived expressions (17) and (22). Monte Carlo simulations are presented to confirm analytical results. It is well-known that the intensity of atmospheric turbulence is determined by the previously defined Rytov variance. We utilize parameter values from [6] − [8], which present some experimental measurements. In order to determine the pointing errors strength, we use jitter standard deviation. Following values of some considered parameters: the radius of a circular detector aperture *a*=5 cm; wavelength employed in FSO link λ=785 nm; FSO link distance *L*=1 km; normalized average optical power of the FSO hop $\Omega + 2b_0 = 1$.

Considering dual-branch FSO system with EGC, the average BER in the function of each branch average electrical SNR ($\mu_1 = \mu_2$), is presented in Fig. 1. Lower values of the normalized jitter standard deviation lead to the better performance. It means that positioning of laser beam is very good. Misalignment between FSO apertures is minor, which reflects in lower values of the average BER. On the other hand, when misalignment between laser and photodetector is great, and there are intense optical beam vibrations, the system performance are worsen due to greater values of the normalized jitter standard deviation.

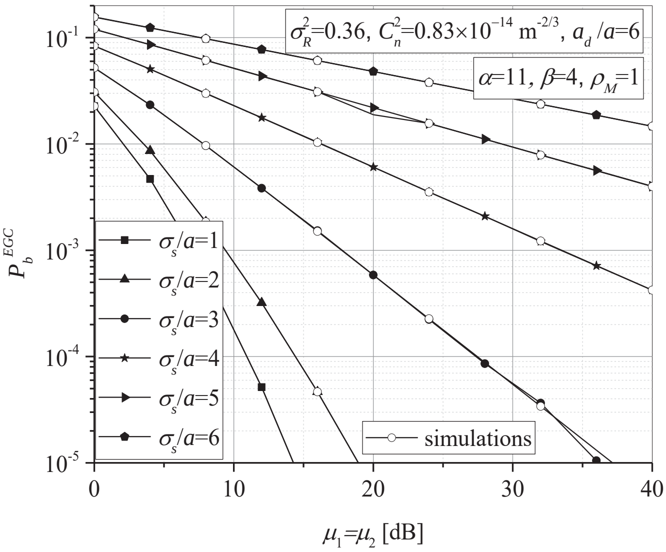

Fig. 1. Average BER of the dual-branch FSO system for different values of the pointing errors standard deviation

Fig. 2 presents the average BER dependence on average electrical SNR of the SISO FSO system. The atmospheric turbulence intensity is determined by results given in [7], which presents experimental measurements performed in University of Waseda, Japan, on the 15th October, 2009. The value of the Rytov variance is $\sigma_R^2 = 0.36$, and the refractive index structure parameter takes a value $C_n^2 = 0.83\times10^{-14}$ $m^{-2/3}$. Also, different pointing errors strength are assumed: $\sigma_s/a = 1$ and $\sigma_s/a = 3$. The parameter ρ_M, which represents the amount of the scattering power coupled to the LOS component, has strong impact on the average BER performance, since the Rytov variance is the same. The greater the parameter ρ_M is, the system performance is better. It means that the amount of the power scattered to the receiver by the off-axis eddies, i.e., power of C_{GS}, is lower, which reflects in the improved BER performance. When $\rho_M = 1$, the FSO system is affected by Gamma-Gamma atmospheric turbulence, which results in the best performance in Fig. 2. It is also observed that the influence of the component C_{GS} on system performance is more dominant in weak pointing error.

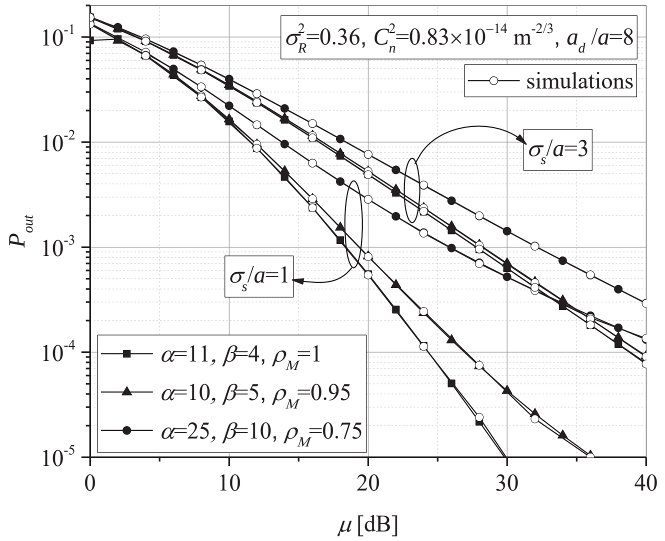

Fig. 2. Average BER of the SISO FSO system for different values of the parameter ρ_M

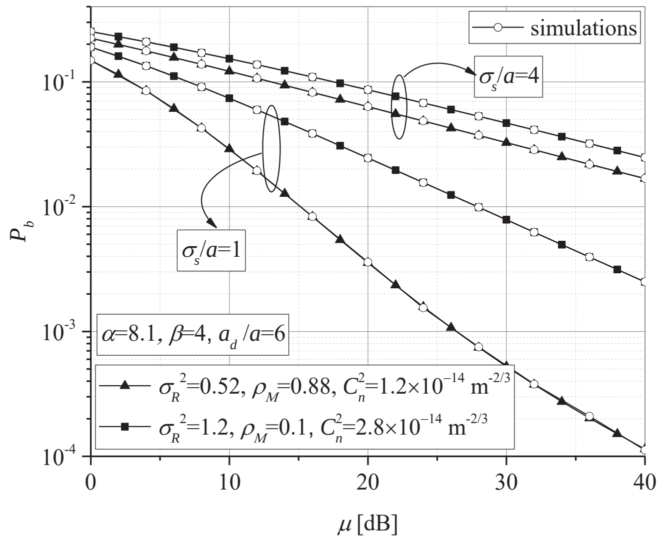

Fig. 3. Average BER of the SISO FSO system for different intensity of the atmospheric turbulence

Based on measurement results presented in [6, Table I], Fig. 3 shows the average BER versus μ, for different intensity of atmospheric turbulence, and various pointing errors conditions. System performs better when $\sigma_R^2 = 0.52$, in sunrise, when atmospheric turbulence is weak. When pointing error is poor, intensity of atmospheric turbulence has greater impact on the average BER.

Fig. 4 presents the average BER dependence on average electrical SNR for different pointing errors strength. The SISO FSO system is observed, as well as dual-branch SIM-BSPK FSO system with EGC receiver. Significant system improvement is noticed with implementation of diversity technique. This improvement is more pronounced in the case when jitter standard deviation is lower, or in other word, when conditions for FSO signal transmission are favorable.

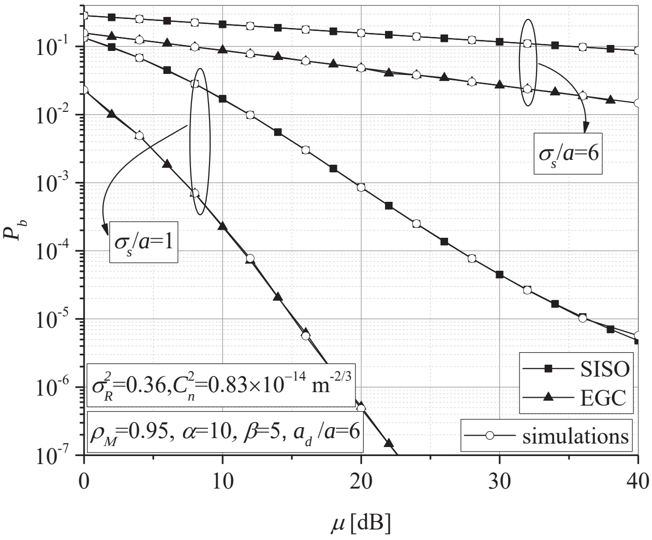

Fig. 4. Average BER of both SISO and dual-branch FSO system

V. CONCLUSION

In this paper, the average BER analysis of the SIM-BPSK FSO system has been presented, considering that M distribution is used for modeling the intensity fluctuations of the optical signal caused by atmospheric turbulence. Pointing errors effect has been also taken into account. As a way to mitigate atmospheric turbulence and pointing errors effects, dual-branch FSO system employing EGC receiver has been also observed. Beside closed-form average BER expression for SISO FSO channel, the average BER expression for dual-branch EGC FSO system has been also derived in integral form. Using derived expressions, numerical results have been presented and confirmed by Monte Carlo simulations. The impact of different atmospheric turbulence and pointing errors parameters on the average BER performance has been observed.

Based on presented results, it has been concluded that atmospheric turbulence and pointing errors can seriously deteriorate system performance. The jitter standard deviation determines the pointing errors strength. When its value is low, displacement of the optical beam is minor, and positioning of the FSO apertures is good. This results in better system performance. The component scattered to the receiver by the off-axis eddies, is an important factor in determination of the average BER, especially when the pointing error is weak.

Also, the intensity of atmospheric turbulence can extremely weaken signal transmission, performance, mainly when the pointing error effect is poor. Employing two photodetector and EGC at the receiver leads to the further system performance. Still, with worsening conditions of the FSO signal transmission, the achieved gain is considerably lower. Hence, the positioning and design of the FSO apertures is an important task, which determine the quality of the system performance.

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